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MECHANICS;
OR, THE
DOCTRINE
OF
MOTION.

Emerson (W.) K

COMPREHENDING,

I. The general Laws of Motion.

II. The Descent of Bodies perpendicularly, and down inclined Planes, and alſo in Curve Surfaces. The Motion of Pendulums.

III. Centers of Gravity. The Equilibrium of Beams of Timber, and their Forces and Directions.

IV. The Mechanical Powers.

V. The comparative Strength of Timber, and its Stress. The Powers of Engines, their Motion, and Friction.

VI. Hydrostatics and Pneumatics.

> *Da veniam ſcriptis, quorum non gloria nobis*
> *cauſa, ſed utilitas officiumque, fuit.*
> Ovid de Ponto III.

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G.R III

THE

PREFACE.

HAVING ſome years ſince written a large book of Mechanics, which is ſold by Meſſ^{rs}. Robinſon, and Co. in Pater-noſter-Row. I have here given a ſhort abſtract of that book, as it falls properly into this courſe; and eſpecially as that branch of ſcience, is of ſuch extenſive uſe in the affairs of human life. I do not in the leaſt, deſign this to interfere with the other book, being rather an introduction to it, as it explains ſeveral things in it more at large; particularly in the firſt ſection, as being of univerſal extent and uſe; and likewiſe in ſeveral other parts of the book, eſpecially ſuch as have been objected to by ignorant writers. I have alſo added ſeveral things not mentioned in the other book, which are more ſimple and eaſy, and more proper for learners. So that this ſhort treatiſe may be looked upon as an introduction to the other book, and will doubtleſs facilitate the reading of it. As to the higher and more difficult matters, as few care to trouble their heads about them, I have ſaid little of them here, being not ſo proper for an introduction. To mention one or two things; I had taken a great deal of pains to find out the true form of a bridge, that ſhall be the ſtrongeſt, and of a ſhip that ſhall ſail the faſteſt; both upon principles that I know to be as certain and demonſtrative as the Elements of Euclid; both theſe you have in the other book. But, as we have no occaſion in England, for *the ſtrongeſt bridges, or the ſwifteſt ſhips; Mathematicians, for the future, may find ſomething elſe to do, than run into ſuch perplext and uſeleſs diſquiſitions. For indeed when any of theſe grand things are to be performed, they generally fall into the hands of ſuch people, as know little of the nature of them. They perhaps know how to lay down upon paper, the plan of a deſign, by rule and compaſs, and to do ſeveral problems in practical geometry; and not much more. As if this was an adequate qualification, for conducting ſuch magnificent works, as coſt many thouſand pounds to execute. So that theſe things, inſtead of being conſtructed by the rules of art; they are too often done by fancy, without any true rules.*

But in this there is no great wonder, conſidering how few people ſtudy this art; and among theſe that do, how few are competent judges. For even among thoſe that preſume to write about it, it is ſurprizing to ſee what miſtakes they daily run into. One denies the compoſition and reſolution of forces; another cannot be ſatisfied with ſome of the mechanical powers, not even ſo ſimple a thing as the wedge; all owing to the wrong notions they have imbibed. And in conſequence of this, will be either condemning the art itſelf, or criticiſing on other writers, whom they do not underſtand.

As to what I have written on this ſubject; I have all along given the demonſtrations of the ſeveral things I have handled; and I expect that *to be my teſt, as to the truth or falſhood thereof. And by this teſt I leave them to be tried by any judicious, honeſt reader; who is a lover of truth, and a promoter of ſcience.*

W. Emerſon.

MECHA-

MECHANICS.

DEFINITIONS.

1. *MECHANICS* is a ſcience, which treats of the forces, motions, velocities, and in general, of the actions of bodies upon one another. It teaches how to move any given weight with any given power; how to contrive engines to raiſe great weights, or to perform any kind of motion.

2. *Body* is the maſs or quantity of matter; an *elaſtic body* is that which yields to a ſtroke, and recovers its figure again. But if not, 'tis called an *unelaſtic body*.

3. *Denſity* is the proportion of the quantity of matter in any body, to the quantity of matter in another body of the ſame bigneſs.

4. *Force* is a power exerted on a body to move it. If it act inſtantaneouſly, 'tis called *Percuſſion*, or *impulſe*. If conſtantly, 'tis an *accelerative force*.

5. *Velocity* is a property of motion, by which a body paſſes over a certain ſpace in a certain time. And is greater or leſſer, as it paſſes over a greater or leſſer ſpace in a certain time as ſuppoſe a ſecond.

6. *Motion* is a continual and ſucceſſive change of place. If the body moves equally, 'tis called *equable* or *uniform motion*. If it increaſes or decreaſes, 'tis called *accelerated* or *retarded motion*. When it is compared with ſome body at reſt, 'tis called *abſolute*

lute motion. But when compared with others in motion, it is called *relative motion.*

7. *Direction* of motion is the courſe or way the body tends, or the line it moves in.

8. *Quantity* of motion, is the motion a body has, conſidered both in regard to its velocity and quantity of matter. This is alſo called the *Momentum* of a body.

9. *Vis inertiæ*, is the innate force of matter, by which it reſiſts any change, ſtriving to preſerve its preſent ſtate of reſt or motion.

10. *Gravity* is that force wherewith a body endeavours to fall downwards. It is called *abſolute gravity* in empty ſpace; and *relative gravity* when immerſed in a fluid.

11. *Specific gravity*, is the greater or leſſer weight of bodies of the ſame magnitude, or the proportion between their weights. This proceeds from the natural denſity of bodies.

12. *Center of gravity*, is a certain point of a body; upon which, the body when ſuſpended, will reſt in any poſition.

13. *Center of motion*, is a fixed point about which a body moves. And the *axis of motion* is a fixed line it moves about.

14. *Power* and *weight*, when oppoſed to one another, ſignify the body that moves another, and the other which is moved. The body which begins and communicates motion is the *power*; and that which receives the motion, is the *weight.*

15. *Equilibrium* is the balance of two or more forces, ſo as to remain at reſt.

16. *Machine* or *Engine*, is any inſtrument to move bodies, made of levers, wheels, pullies, &c.

17. *Mechanic powers*, are the ballance, lever, wheel, pulley, ſcrew and wedge.

18. *Streſs* is the effect any force has to break a beam, or any other body; and *ſtrength* is the

reſiſtance it is able to make againſt any ſtraining force.

19. *Friction* is the reſiſtance which a machine ſuffers, by the parts rubbing againſt one another.

POSTULATA.

1. That a ſmall part of the ſurface of the earth may be looked upon as a plane. For tho' the earth be round, yet ſuch a ſmall part of it as we have any occaſion to conſider, does not ſenſibly differ from a plane.

2. That heavy bodies deſcend in lines parallel to one another. For tho' they all tend to a point which is the center of the earth, yet that center is at ſuch a diſtance that theſe lines differ inſenſibly from parallel lines.

3. The ſame body is of the ſame weight in all places on or near the earth's ſurface. For the difference is not ſenſible in the ſeveral places we can go to.

4. Tho' all matter is rough, and all engines imperfect; yet for the eaſe of calculation, we muſt ſuppoſe all planes perfectly even; all bodies perfectly ſmooth; and all bodies and machines to move without friction or reſiſtance; all lines ſtreight and inflexible, without weight or thickneſs; cords extremely pliable, and ſo on.

AXIOMS.

1. Every body endeavours to remain in its preſent ſtate, whether it be at reſt, or moving uniformly in a right line.

2. The alteration of motion by any external force is always proportional to that force, and in direction of the right line in which the force acts.

3. Action and re-action, between any two bodies, are equal and contrary.

4. The motion of any body is made up of the ſum of the motions of all the parts.

5 The weights of all bodies in the ſame place, are proportional to the quantities of matter they contain, without any regard to their figure.

6. The vis inertiæ of any body, is proportional to the quantity of matter.

7. Every body will deſcend to the loweſt place it can get to.

8. Whatever ſuſtains a heavy body, bears all the weight of it.

9. Two equal forces acting againſt one another in contrary directions; deſtroy one anothers effects. And unequal forces act only with the difference of them.

10. When a body is kept in equilibrio; the contrary forces in any line of direction are equal.

11. If a certain force generate any motion; an equal force acting in a contrary direction, will deſtroy as much motion in the ſame time.

12. If a body be acted on by any power in a given direction. It is all one in what point of that line of direction, the power is applied.

13. If a body is drawn by a rope, all the parts of the rope are equally ſtretched. And the force in any part acts in direction of that part. And it is the ſame thing whether the rope is drawn out at length, or goes over ſeveral pullies.

14 If ſeveral forces at one end of a lever, act againſt ſeveral forces at the other end; the lever acts and is acted on in direction of its length.

SECT. I.

The General Laws of MOTION.

PROP. I.

THE quantities of matter in all bodies, are in the compound ratio of their magnitudes and denſities.

For (Def. 3.) in bodies of the ſame magnitudes, the quantities of matter will be as the denſities. Increaſe the magnitude in any ratio, and the quantity of matter is increaſed in the ſame ratio. Conſequently the quantity of matter is in the compound ratio of the denſity and magnitude.

Cor. 1. *In two ſimilar bodies, the quantities of matter are as the denſities, and cubes of the diameters.* For the magnitudes of bodies are as the cubes of the diameters.

Cor. 2. *The quantities of matter are as the magnitudes and ſpecific gravities.*

For (by Def. 3. and 11.) the denſities of bodies are as their ſpecific gravities.

PROP. II.

The quantities of motion in all moving bodies, are in the compound ratio of the quantities of matter and the velocities.

For if the velocities be equal, the quantities of motion will manifeſtly be as the quantities of matter. Increaſe the velocity in any ratio, and the quantity of motion will be increaſed in the ſame ratio,

ratio. Therefore it follows univerſally, that the quantities of motion are in the compound ratio of the velocities and quantities of matter.

Cor. *Hence if the body be the ſame, the motion is as the velocity. And if the velocity be the ſame, the motion is as the body or quantity of matter.*

PROP. III.

In all bodies moving uniformly, the ſpaces deſcribed, are in the compound ratio of the velocities and the times of their deſcription.

For in any moving body, the greater the velocity, the greater is the ſpace deſcribed; that is, the ſpace will be as the velocity. And in twice or thrice the time, &c. the ſpace will be twice or thrice as great; that is, the ſpace will be increaſed in proportion to the time. Therefore univerſally the ſpace is in the compound ratio of the velocity, and the time of deſcription.

Cor. 1. *The time of deſcribing any ſpace, is as the ſpace directly and velocity reciprocally; or as the ſpace divided by the velocity. And if the velocity be the ſame, the time is as the ſpace. And if the ſpace be the ſame, the time is reciprocally as the velocity.*

Cor. 2. *The velocity of a moving body, is as the ſpace directly, and time reciprocally; or as the ſpace divided by the time. And if the time be the ſame, the velocity is as the ſpace deſcribed. And if the ſpace be the ſame, the velocity is reciprocally as the time of deſcription.*

PROP.

PROP. IV.

The motion generated by any momentary force, or by a ſingle impulſe, is as the force that generates it.

For if any force generates any quantity of motion; double the force will produce double the motion; and treble the force, treble the motion, and ſo on. If a body ſtriking another, gives it any motion, twice that body ſtriking the ſame, with the ſame velocity, will give it twice the motion, and ſo the motion generated in the other will be as the force of percuſſion.

Cor. 1. *Hence the forces are in the compound ratio of the velocities and quantities of matter.*

For (Prop. II.) the motions are as the quantities of matter multiplied by the velocities.

Cor. 2. *The velocity generated, is as the force directly, and quantity of matter reciprocally. Therefore if the bodies are equal, the velocities are as the forces. And if the forces are equal, the velocities are reciprocally as the bodies.*

Cor. 3. *The quantity of matter, is as the force directly, and velocity reciprocally. And therefore if the velocities be equal, the bodies are as the forces. And if the forces be equal, the bodies are reciprocally as the velocities.*

PROP. V.

The quantity of motion generated, by a conſtant and uniform force, is in the compound ratio of the force and time of acting.

For the motion generated in any given time will be proportional to the force that generates it;

and in twice that time, the motion will be double by the ſame force; and in thrice that time it will be treble; and ſo every part of time adds a new quantity of motion equal to the firſt; and therefore the whole motion, will be as the force, and the whole time of acting.

Cor. 1. *The motion loſt in any time, is in the compound ratio of the force and time.*

Cor. 2. *The velocity generated (or deſtroyed) in any time, is as the force and time directly, and quantity of matter reciprocally. The ſame is true of the increaſe or decreaſe of velocity.*

For the motion, that is (Prop. II.) the body multiplied by the velocity, is as the force and time. And therefore the velocity is as the force and time directly, and the body reciprocally.

Cor. 3. *Hence if the force be as the quantity of matter, the velocity is as the time. Or if the force and quantity of matter be given, the velocity is as the time.*

And if the time and quantity of matter be given, the velocity is as the force.

And if the force and time be given, the velocity is reciprocally as the matter.

Cor. 4. *The time is as the quantity of matter and velocity directly, and the force reciprocally. Therefore,*

If the force and velocity be given, the time is as the quantity of matter.

If the quantity of matter and velocity be given, the time is reciprocally as the force.

Cor. 5. *The force is as the quantity of matter and velocity directly, and the time reciprocally. Whence,*

If the velocity is at the time, or if the velocity and time be given, then the force is as the quantity of matter.

And if the velocity and quantity of matter be given, the force is reciprocally as the time.

Cor. 6. *The quantity of matter is as the force and time directly, and the velocity reciprocally. Therefore, if the force and time be given, the quantity of matter is reciprocally as the velocity.*

Cor. 7. *Hence alſo if the body be given, the velocity is in the compound ratio of the force and time.*

And if the force be given, the time is in the compound ratio of the matter and velocity, or as the quantity of motion.

PROP. VI.

If a given body is urged by a conſtant and uniform force; the ſpace deſcribed by the body from the beginning of the motion, will be as the force and ſquare of the time.

Suppoſe the time divided into an infinite number of equal parts or moments. Then in each of theſe moments of time, the ſpace deſcribed (Prop. III. Cor. 2.) will be as the velocity gained; that is, (by Cor. 7. Prop. V.) as the force and time from the beginning. And the ſum of all the ſpaces, or the whole ſpace deſcribed, will be as the force and the ſum of all the moments of time from the beginning. Therefore put $t =$ the whole time, and the whole ſpace deſcribed will be as the force and ſum of the times 1, 2, 3, 4, &c. to t. But the ſum of the arithmetic progreſſion $1 + 2 + 3 + 4$ . . . to $t = \frac{t+1}{2} \times t = \frac{1}{2} tt$, becauſe t is infinite or conſiſts of an infinite number of moments. Therefore the whole ſpace deſcribed will be as the force and $\frac{1}{2} tt$; that is, (becauſe $\frac{1}{2}$ is a given quantity), as the force and the ſquare of the time of deſcription.

Cor. 1. *If a body is impelled by a conſtant and uniform force; the ſpace deſcribed from the beginning of the motion, is as the velocity gained, and the time of moving.*

For the ſpace is as the force and ſquare of the time, or as the force × time × time. But (Cor. 7. Prop. V.) the force × time is as the velocity; therefore the ſpace which is as the force × time × time, is as the velocity × time.

Cor. 2. *If a body urged by any conſtant and uniform force, deſcribes any ſpace; it will deſcribe twice that ſpace in the ſame time, by the velocity acquired.*

For the ſum of all the ſpaces deſcribed by that force, $1 + 2 + 3$ &c. to t, was ſhewn to be $\frac{1}{2}tt$. But the ſum of all the ſpaces deſcribed by the laſt velocity, will be $t + t + t$ &c. to t terms, whoſe ſum is tt. But tt is double to $\frac{1}{2}tt$; that is, the ſpace deſcribed by the laſt velocity, is double the ſpace deſcribed by the accelerating force.

Cor. 3. *Univerſally in all bodies urged by any conſtant and uniform forces; the ſpace deſcribed is as the force and ſquare of the time directly, and the quantity of matter reciprocally.*

For (Cor. 1.) the ſpace is as the time and velocity. But (Prop. V. Cor. 2.) the velocity is univerſally as the force and time directly, and quantity of matter reciprocally. Therefore the ſpace is as the ſquare of the time and the force directly, and matter reciprocally; whence,

Cor. 4. *The product of the force, and ſquare of the time, is as the product of the body and ſpace deſcribed.*

Cor. 5. *The product of the force and time, is as the product of the quantity of matter and velocity.*

For (Prop. V.) the product of the force and time, is as the motion; that is, as the body and velocity. Fig.

Cor. 6. *The product of the body, and ſquare of the velocity, is as the product of the force and the ſpace deſcribed.*

For (Cor. 5.) the product of the body and velocity, is as the force and time. Therefore, the body × velocity ſquare, is as the force × time × velocity; but time × velocity is as the ſpace (by Prop. III.); therefore body × velocity ſquare is as the force × ſpace.

SCHOLIUM.

If any quantity or quantities are given, they muſt be left out. And ſuch quantities as are proportional to each other muſt be left out. For example, if the quantity of matter be always the ſame; then (Cor. 3.) the ſpace deſcribed is as the force and ſquare of the time. And if the matter be proportional to the force, as all bodies are in reſpect to their gravity; then (Cor. 6.) the ſpace deſcribed is as the ſquare of the velocity. Or if the ſpace deſcribed be always proportional to the body; then (Cor. 6.) the force is as the ſquare of the velocity. Again, if the body be given, then (Cor. 4.) the ſpace is as the force and ſquare of the time. And if both the quantity of matter and the force be given, the ſpace deſcribed is as the ſquare of the time. And ſo of others.

PROP. VII.

If ABCD *be a parallelogram; and if a body at* A, *be acted upon ſeparately by two forces, in the directions* AB *and* AC, *which would cauſe the body to be carried thro' the ſpaces* AB, AC *in the ſame time. Then both forces acting at once, will cauſe the body to be carried thro' the diagonal* AD *of the parallelogram.* 1.

Let the line AC be ſuppoſed to move parallel to itſelf;

Fig. 1. itſelf; whilſt the body at the ſame time moves from A, along the line AC or *bg*, and comes to *d* at the ſame inſtant, that AC comes to *bg*. Then ſince the lines AB, AC, are deſcribed in the ſame time; and A*b*, A*d*, are alſo deſcribed in the ſame time. Therefore, as the motions are uniform, it will be, A*b* : *bd* : : AB : BD; and therefore A*d*D is a ſtreight line, coinciding with the diagonal of the parallelogram.

Cor. 1. *The three forces in the directions* AB, AC, AD, *are reſpectively as the lines* AB, AC, AD.

Cor. 2. *Any ſingle force* AD *denoted by the diagonal of a parallelogram, is equivalent to two forces denoted by the ſides* AB, AC.

Cor. 3. *And therefore any ſingle force* AD *may be reſolved into two forces, an infinite number of ways, by drawing any two lines* AB, BD, *for their quantities and directions.*

SCHOLIUM.

This practice of finding two forces equivalent to one, or dividing one force into two; is called the *compoſition* and *reſolution* of forces.

PROP. VIII.

2. *If three forces* A, B, C, *keep one another in equilibrio; they will be proportional to three ſides of a triangle, drawn parallel to their ſeveral directions,* DI, CI, CD.

Produce AD to I, and BD to H, and compleat the parallelogram DICH; then (Prop. VII.) the force in direction DC, is equal to the forces DH, DI, in the directions DH, DI. Take away the force DC and putting the forces DH, DI equal

thereto; and the equilibrium will ſtill remain. Fig.
Therefore (Ax. 10.) DI is equal to the force A oppoſite to it; and DH or CI equal to its oppoſite force B. And as CD repreſents the force C, the three forces A, B, and C will be to one another as DI, CI, and CD. 2.

Cor. 1. *Hence if three forces acting againſt one another, keep each other in equilibrio; theſe forces will be reſpectively as the three ſides of a triangle drawn perpendicular to their lines of direction; or making any given angle with them, on the ſame ſide.*

For this triangle will be ſimilar to a triangle whoſe ſides are parallel to the lines of direction.

Cor. 2. *If three active forces* A, B, C *keep one another in equilibrio; they will be reſpectively as the ſines of the angles, which their lines of direction paſs through.*

For A, B, C are as DI, CI, CD; that is, as S.DCI, S.CDI, and S.DIC. But S.DCI = S.CDH = S.CDB. And S.CDI = S.CDA. Alſo S.DIC = S.BDI = S.BDA.

Cor. 3. *If ever ſo many forces acting againſt one another, are kept in equilibrio, by theſe actions; they may be all reduced to two equal and oppoſite ones.*

For any two forces may, by compoſition, be reduced to one force acting in the ſame plane. And this laſt force, and any other, may likewiſe be reduced to one force acting in the plane of theſe; and ſo on, till they all be reduced at laſt to the action of two equal and oppoſite ones.

PROP. IX.

If a body impinges or acts againſt any plain ſurface; it exerts its force in a line perpendicular to that ſurface. 3.

Let the body A moving in direction AB, with a given velocity, impinge on the ſmooth plain FG at the

Fig. 3. the point B. Draw AC parallel, and BC perpendicular to FG; and let AB repreſent the force of the moving body. The force AB is, by the reſolution of forces, equivalent to AC and CB. The force AC is parallel to the plain, and therefore has no effect upon it; and therefore the ſurface FG is only acted upon by the force CB, in a direction perpendicular to the ſurface FG.

Cor. 1. *If a body impinges upon another body with a given velocity; the quantity of the ſtroke is as the ſine of the angle of incidence.*

For the abſolute force is AB, and the force acting on the ſurface FG is CB. But AB : CB : : rad : S.CAB or ABF.

Cor. 2. *If an elaſtic body* A *impinges upon a hard or elaſtic plane* FG; *the angle of reflexion will be equal to the angle of incidence.*

For if AD be parallel to FG; the motion of A in direction AD parallel to the plane, is not at all changed by the ſtroke. And by the elaſticity of one or both bodies, the body A is reflected back to AD in the ſame time it moved from A to B; let it paſs to D; then will AC = CD, being deſcribed in equal times; conſequently the angle ABC = angle CBD; and therefore the angle DBG = angle ABF.

Cor. 3. *If a non-elaſtic body ſtrikes another non-elaſtic body; it loſes but half the motion, that it would loſe, if the bodies were elaſtic.*

For non-elaſtic bodies only ſtop, without receding from one another; but elaſtic bodies recede with the ſame velocity.

PROP. X.

The ſum of the motions of two or more bodies, in any direction towards the ſame part, cannot be changed by any action of the bodies upon each other.

Here I reckon progreſſive motions affirmative; and regreſſive ones negative, and to be deducted out of the reſt to get the ſum.

1. If two bodies move the ſame way; ſince action and re-action are equal and contrary, what one body gains the other loſes; and the ſum remains the ſame as before. And the caſe is the ſame, if there were more bodies.

2. If bodies ſtrike one another obliquely; they will act on one another in a line perpendicular to the ſurface acted on. And therefore by the law of action and re-action there is no change made in that direction.

3. And in a direction parallel to the ſtriking ſurface, there is no action of the bodies, therefore the motion remains the ſame in that direction. Whence the motions will remain the ſame in any one line of direction.

Cor. 1. *Motion can neither be increaſed nor decreaſed, conſidered in any one direction; but muſt remain invariably the ſame for ever.*

This follows plainly from this Prop. for what motion is gained to one (by addition), is loſt to another body (by ſubtraction); and ſo the total ſum remains the ſame as before.

SCHOLIUM.

This Prop. does not include or meddle with ſuch motions as are eſtimated in all directions. For upon this ſuppoſition, motion may be increaſed or decreaſed

Fig. decreaſed an infinite number of ways. For example, if two equal and non-elaſtic bodies, with equal velocities, meet one another; both their motions are deſtroyed by the ſtroke. Here at the beginning of the motion, they had both of them a certain quantity of motion, but to be taken in contrary directions; but after the ſtroke they had none.

PROP. XI.

The motion of bodies included in a given ſpace, is the ſame, whether that ſpace ſtands ſtill; or moves uniformly in a right line.

For if a body be moving in a right line, and any force be equally impreſſed both upon the body and the line it moves in; this will make no alteration in the motion of the body along the right line. And for the ſame reaſon, the motions of any number of bodies in their ſeveral directions will ſtill remain the ſame; and their motions among themſelves will continue the ſame, whether the including ſpace is at reſt, or moves uniformly forward. And ſince the motions of the bodies among themſelves; that is, their relative motions remain the ſame, whether the ſpace including them be at reſt, or has any uniform motion. Therefore their mutual actions upon one another, muſt alſo remain the ſame in both caſes.

SECT.

SECT. II.

The perpendiculur deſcent of heavy bodies, their deſcent upon inclined Planes, and in Curve Surfaces. The Motion of Pendulums.

PROP. XII.

THE velocities of bodies, falling freely by their own weight, are as the times of their falling from reſt.

For ſince the force of gravity is the ſame in all places near the earth's ſurface (by Poſt. 3.), and this is the force by which bodies deſcend. Therefore the falling body is urged by a force which acts conſtantly and equally; and therefore (by Cor. 3. Prop. V.) the velocity generated in the falling body in any time, is as the time of falling.

Cor. 1. *If a body be thrown directly upwards with the ſame velocity it falls with; it will loſe all its motion in the ſame time.*

For the ſame active force will deſtroy as much motion in any time, as it can generate in the ſame time.

Cor. 2. *Bodies deſcending or aſcending gain or loſe equal velocities in equal times.*

PROP. XIII.

In bodies falling freely by their own weight; the ſpaces deſcribed in falling from reſt, are as the ſquares of the times of falling.

For ſince gravity is ſuppoſed to be the ſame in all places near the earth. Therefore the falling

body

Fig. body will be acted on by a force which is conſtantly the ſame; and therefore (by Prop. VI.) the ſpaces deſcribed, from the beginning of the motion, or ſince their falling from reſt, will be as the ſquares of the times of falling.

Cor. 1. *The ſpaces deſcribed by falling bodies are as the ſquares of the velocities.*

For (Prop. XII.) the velocities are as the times of falling.

Cor. 2. *The ſpaces deſcribed by falling bodies, are in the compound ratio of the times, and the velocities acquired by falling.*

Cor. 3. *If a body falls through any ſpace, and move afterwards with the velocity gained in falling; it will deſcribe twice that ſpace in the time of its falling.*

Cor. 4. *A body projected upward with the velocity it gained in falling, will aſcend to the ſame height it fell from.*

SCHOLIUM.

All theſe things would be true if it was not for the reſiſtance of the air, which will retard their motion a little. In very ſwift motions, the reſiſtance of the air has a very great effect in deſtroying the motions of bodies.

PROP. XIV.

4. *If a heavy body* W *be ſuſtained upon an inclined plane* AC, *by a power* F *acting in direction* WF *parallel to the plane; and if* AB *be parallel to the horizon and* BC *perp. to it. Then if the length* AC *denotes the weight of the body, the hight* CB *will denote the power at* F *which ſuſtains it, and the baſe* AB *the preſſure againſt the plane.*

Draw BD perpendicular to AC, then CB will be the direction of gravity, DC parallel to WF will be the direction of the force at F, and DB the direction of the preſſure (by Prop. IX). Therefore (by Prop. VIII.) the weight of the body, the power at F, and the preſſure; will be reſpectively as BC, CD and DB. But the triangles ABC, BDC are ſimilar, and therefore BC, CD and DB are reſpectively as AC, CB and BA. Therefore the weight, power and preſſure, are as the lines AC, CB and AB. Fig. 4.

Cor. 1. *The weight of the body, the power* F *that ſuſtains it on the plane, and the preſſure againſt the plane; are reſpectively as radius, the ſine and coſine of the planes elevation above the horizon.*

For AC, CB and AB are to one another, as radius, ſine of CAB, and ſine of ACB.

Cor. 2. *The power that urges a body* W *down the inclined plane is* $= \frac{CB}{AC} \times$ *weight of* W. *Hence,*

Cor. 3. *If a priſmatic body whoſe length is* AC *lie upon the inclined plane* AC; *it is urged down the plane with a force equal to the weight of the priſm of the length* CB.

Cor. 4. *If there be two planes of the ſame hight, and two bodies be laid on them proportional to the lengths of theſe planes; the tendency down the planes will be equal in both bodies.*

PROP. XV.

If AC *be an inclined plane,* AB *the horizon,* BC *perp. to* AB. *And if* W *be a heavy body upon the plane, which is kept there by the power* P *acting in direction* WP. *Draw* BDE *perp. to* WP. *Then the weight* W, *the power* P, *the preſſure againſt the plane; will be reſpectively as* AB, DB *and* AD. 5.

For AB being a horizontal plane is perpendicular to the action of gravity; and BD is perp. to

the

Fig. 5. the direction of the power P; and AD is the plane, which is perp. to the direction of the preſſure againſt that plane. Therefore (Cor. 1. Prop. VIII.) the weight of the body, the power P, and the preſſure; are as AB, BD and AD. And if the direction of the power WP be under the plane, the proportion will be the ſame, as long as BD is perpendicular to WP.

Cor. 1. *The weight of the body* W, *is to the power* P *that ſuſtains it* : : *as coſine of the angle of traction* CWP : *to the ſine of the plane's elevation* CAB.

For the weight : power : : AB : BD : : S.ADB or WDE : S.BAD : : coſ DWE or CWP : S.BAC, where the angle CWP made by the plane and direction of the power is called the angle of Traction.

Cor. 2. *Hence it is the ſame thing as to the power and weight, whether the line of direction is above or below the plane, provided the angle of traction be the ſame. For an equal power will ſuſtain the weight in both caſes.*

Cor. 3. *The weight of the body* : *is to the preſſure againſt the plane* : : *as the coſine of the angle of traction* CWP : *to the coſine of* BNP, *the direction of the power above the horizon.*

For the weight W : preſſure : : AB : AD : : S.ADB : S.ABD : : S.ADE : : S.NBE : : coſ. EWD or PWC : coſ. BNP.

Cor. 4. *Hence the preſſure againſt the plane is greater when the direction of the power is below the plane, the weight remaining the ſame.*

SCHOLIUM.

Altho' the power has the ſame proportion to the weight, when the angle of traction is the ſame; whether the direction of the power be above or below

Fig. 5. below the plane Yet, ſince the preſſure upon the plane is greater, when the line of direction is below the plane. Therefore in practice, when a weight is to be drawn up hill, if it is to be done by a power whoſe direction is below the plane, the greater preſſure in this caſe will make the carriage ſink deeper into the earth, &c. and for that reaſon will require a greater power to draw it up, than when the line of direction is above the plane.

PROP. XVI.

6. *If a weight* W *upon an inclined plane* AC, *be in equilibrio with another weight* P *hanging freely*; *then if they be ſet a moving, their perpendicular velocities in that place, will be reciprocally as their quantities of matter.*

Take WA a very ſmall line upon the plane AC; draw AB parallel to the horizon, and BC perp. to it. Draw AF, and WR, BE perpendicular to it; and WT, DV perp. to AB. Let W deſcend thro' the ſmall line WA upon the plane, then P will aſcend a hight equal to AR perpendicularly; and WT will be the perpendicular deſcent of W. The triangles AWR and ADE are ſimilar; and likewiſe the triangles AWT and ADV. Therefore WT : DV : : AW : : WD : : WR : DE. And alternately, WT : WR : : DV : DE; and WR : AR : : DE : AE; therefore WT : AR : : DV : AE : : (by the ſimilar triangles DBV and AEB) DB : AB : : (Prop. XV.) power P : weight W.

Cor. 1. *If any two bodies be in equilibrio upon two inclined planes*; *their perpendicular velocities will be reciprocally as the bodies.*

Cor. 2. *If two bodies ſuſtain each other in equilibrio, on any planes*; *the product of one body* × *by its perp.*

Fig. *perp. velocity, is equal to the product of the other bo-*
6. *dy × by its perp. velocity.*

PROP. XVII.

7. *If a heavy body runs down an inclined plane* CA; *the velocity it acquires in any time, moving from reſt; is to the velocity acquired by a body falling perpendicularly in the ſame time; as the hight of the plane* CB, *to its length* CA.

The force by which a body endeavours to deſcend on an inclined plane, is to its weight or the force of gravity; as CB to CA (by Prop. XIV.). And as theſe forces always remain the ſame, therefore (Cor. 2. Prop. V.) the velocities generated will be as theſe forces, and the times of acting, directly; and the bodies reciprocally. And ſince the times of acting, and the bodies are the ſame in both caſes, the velocities generated will be as theſe forces; that is, as the hight of the plane CB to its length CA.

Cor. 1. *The velocity acquired by a body running down an inclined plane, is as the time of its moving from reſt.*

Cor. 2. *If a body is thrown up an inclined plane, with the velocity it acquired in deſcending; it will loſe all its motion in the ſame time.*

PROP. XVIII.

7. *If a heavy body deſcends down an inclined plane* CA; *the ſpace it deſcribes from the beginning of the motion, is to the ſpace deſcribed by a body falling perpendicularly in the ſame time; as the hight of the plane* CB, *to its length* CA.

For the force urging the body down the plane is to the force of its gravity, as CB to CA (by Prop. XIV.),

XIV.), which forces remain conſtantly the ſame. Fig. 7. And ſince (Prop. XVII.) the velocities generated in equal times on the plane, and in the perpendicular, are conſtantly as CB to CA; the ſmall particles of ſpace deſcribed with theſe velocities, in all the infinitely ſmall portions of time, will ſtill be in the ſame ratio; and therefore the ſums of all theſe ſmall ſpaces, or the whole ſpaces deſcribed from the beginning, will be in the ſame conſtant ratio of CB to CA.

Cor. 1. *The ſpace deſcribed by a body falling down an inclined plane, in a given time, is as the ſine of the plane's elevation.*

For if CB be given, and alſo the perp. deſcent; that ſpace will be reciprocally as CA, or directly as S.CAB.

Cor. 2. *The ſpaces deſcribed by a body deſcending from reſt, down an inclined plane, are as the ſquares of the times.*

Cor. 3. *If* BD *be drawn perp. to the plane* CA; *then in the time a body falls perpendicularly thro' the hight* CB, *another body will deſcend thro' the ſpace* CD *upon the plane.*

For by ſimilar triangles CA : CB : : CB : CD.

PROP. XIX.

If AC *is an inclined plane, the time of a body's deſcending thro' the plane* CA, *is to the time of falling perpendicularly thro' the hight of the plane* CB, *as the length of the plane* CA *to the hight* CB. 7.

For if BD is perp. to CA, then (Cor. 2. Prop. XVIII.) ſpace CD : ſpace CA : : ſquare of the time in CD : ſquare of the time in CA : : that is, as the ſquare of the time of deſcending perpendicularly

Fig. cularly in CB (Cor 3. laſt) : ſquare of the time in
7. CA. But CD : CA : : CB^2 : CA^2. Therefore CB^2 : CA^2 : ſquare of the time in CB : ſquare of the time in CA. And CB : CA : : time in CB : time in CA.

Cor. 1. *If a body be thrown upwards on the plane with the velocity acquired in deſcending; it will in an equal time aſcend to the ſame hight.*

Cor. 2 *The times wherein different planes, of the ſame hight, are paſſed over; are as the lengths of the planes.*

Let the planes be CA, CF. Then time in AC : time in CB : : CA : CB. And the time in CB : time in CF : : CB : CF. Therefore *ex equo*, time in AC : time in CF : : CA : CF.

PROP. XX.

8. *If a body falls down an inclined plane, it acquires the ſame velocity as a body falling perpendicularly thro' the hight of the plane.*

Let the body run down the plane CA whoſe hight is CB. Draw DF parallel to AB, and infinitely near it. Then the velocities in DA and FB, may be looked upon as uniform. Now (Prop. XIX.) the times of deſcribing CA and CB, will be as CA and CB. Likewiſe the times of deſcribing CD and CF, will be as CD and CF; that is, as CA and CB. And by diviſion, the difference of the times, or the times of deſcribing DA and FB, will alſo be as CA and CB; that is, as DA and FB. But (from Prop. III.) the velocities are equal when the ſpaces are as the times of deſcription. Therefore velocity at A is equal to the velocity at B.

Cor. 1. *The velocities acquired, by bodies deſcending on any planes, from the ſame hight to the ſame horizontal line, are equal.* Fig. 8.

Cor. 2. *If the velocities be equal at any two equal altitudes* D, F; *they will be equal at all other equal altitudes* A, B.

Cor. 3. *Hence alſo, if ſeveral bodies be moving in different directions, thro' any ſpace contained between two parallel planes; and be acted on by any force, which is equal at equal diſtances from either plane. Then if their velocities be equal at entering that ſpace; they will alſo be equal at emerging out of it.*

For dividing that ſpace into infinitely ſmall parts by parallel planes. Then the force between any two planes may be ſuppoſed uniform; and ſuppoſing DF, AB to repreſent two of theſe planes, then (by Cor. 2.) the velocities at D and F being equal; the velocities at A and B will be equal; that is, the velocities at entering the firſt part of ſpace being equal, the velocities at emerging out of it, or at entering the ſecond ſpace will be equal. And for the ſame reaſon the velocities at entering the ſecond ſpace being equal, thoſe at emerging out of it into the third, will be equal. And conſequently the velocities at entering into, and emerging out of the third, fourth, fifth, &c. to the laſt will be equal reſpectively.

PROP. XXI.

If a body falls from the ſame hight, thro' any number of contiguous planes AB, BC, CD; *it will at laſt gain the ſame velocity as a body falling perpendicularly from the ſame hight.* 9.

Let FH be a horizontal line, FD perp. to it. Produce the planes BC, DC to G and H. Then (Cor.

Fig. 9. (Cor. 1. Prop. XX.) the velocity at B is the ſame whether the body deſcend thro' AB or GB. And therefore the velocity at C will be the ſame, whether the body deſcends thro' ABC or thro' GC, and this is the ſame as if it had deſcended thro' HC. And conſequently it will have the ſame velocity at D, in deſcending thro' the planes ABCD, as in deſcending thro' the plane HD; that is, (Prop. XX.) as it has in deſcending thro' the perpendicular FD.

Cor. 1. *Hence a body deſcending along any curve ſurface, will acquire the ſame velocity, as if it fell perpendicularly thro' the ſame hight.*

For let the number of planes be increaſed, and their length diminiſhed ad infinitum, and then ABCD will become a curve. And the velocity acquired by deſcending thro' theſe infinite planes; that is, thro' the curve ABCD, will be the ſame as in falling perpendicularly thro' FD.

Cor. 2. *If a body deſcends in a curve, and another deſcends perpendicularly from the ſame hight. Their velocities will be equal at all equal altitudes.*

Cor. 3. *If a body, after its deſcent in a curve, ſhould be directed upwards with the velocity it had gained; it will aſcend to the ſame hight from which it fell.*

For ſince gravity acts with the ſame force whether the body aſcends or deſcends, it will deſtroy the velocity in the aſcent, in the ſame time it did generate it in the deſcent.

Cor. 4. *The velocity of a body deſcending in any curve, is as the ſquare root of the hight fallen from.*

For it is the ſame as in falling perpendicularly; and in falling perpendicularly, it is as the ſquare root of the hight.

Cor. 5. *If a body, in moving thro' any ſpace* ED, *be acted on uniformly by any force; its velocity at emerging out of it at* D, *will be equal to the ſquare root of the ſum of the ſquares of the velocity at* E *in entering of it, and of the velocity acquired in falling from reſt thro' that ſpace* ED. *And this holds whether the body moves perpendicularly or obliquely.* Fig. 9.

For let the body enter the ſpace ED at E, with the velocity acquired in falling thro' FE. Then (Prop. XIII. Cor. 1.) the ſquare of the velocity at E will be as FE; and the ſquare of the velocity at D, as FD; and the ſquare of the velocity at D falling from E, will be as ED. But FD $=$ FE $+$ ED; therefore the ſquare of the velocity at D (falling thro' FD) $=$ ſquare of the velocity at E $+$ ſquare of the velocity at D (falling thro' ED). And (Cor. 1. of this Prop.) the velocity will be the ſame whether the body deſcends perpendicularly or obliquely.

PROP. XXII.

The times of bodies deſcending thro' two ſimilar parts of ſimilar curves, placed alike, are as the ſquare roots of their lengths. 10.

Let ABCD and *abcd* be two ſimilar curves, and ſuppoſe BC and *bc* to be infinitely ſmall, and ſimilar to the whole; that is, ſo that BC : *bc* : : AB : *ab*. Draw FA parallel to the horizon, and HB, *hb* perp. to it. Then if two bodies deſcend from A and *a* (Cor. 4. Prop. XXI.) the velocities at B and *b* will be as $\sqrt{HB}$ and $\sqrt{hb}$; that is, as $\sqrt{AB}$ and $\sqrt{ab}$, becauſe AB, *ab* are ſimilar parts. Therefore (Prop. III. Cor. 1.) the times of deſcribing BC and *bc*, are as $\frac{BC}{\sqrt{AB}}$ and $\frac{bc}{\sqrt{ab}}$; that is, Fig. 10. as $\frac{AB}{\sqrt{AB}}$ and $\frac{ab}{\sqrt{ab}}$ or as $\sqrt{AB}$ and $\sqrt{ab}$; that is, as $\sqrt{AD}$ and $\sqrt{ad}$, becauſe the curves are ſimilarly divided in B and *b*. After the ſame manner the times of deſcribing any other two ſimilar parts as BC, *bc*, will be as $\sqrt{AD}$ and $\sqrt{ad}$. Therefore by compoſition the times of deſcribing all the BC's, and all the *bc*'s will be as $\sqrt{AD}$ and $\sqrt{ad}$. That is, the time of deſcribing the curve AD to the time of deſcribing the curve *ad*, is as $\sqrt{AD}$ to $\sqrt{ad}$.

Cor. *If two bodies deſcend thro' two ſimilar curves* ABD, *and abd; the axes of the curves* FD, F*d are as the ſquares of the times of their deſcending.*

For $\sqrt{FD} : \sqrt{Fd} :: \sqrt{AD} : \sqrt{ad}$ :: time of deſcending thro' ABD : time of deſcending thro' *abd*. And FD, F*d*, are as the ſquares of the times.

PROP. XXIII.

11. *A body will deſcend thro' any chord of a circle, in the ſame time that another deſcends perpendicularly thro' the diameter.*

Draw the diameter AB perpendicular to the horizon, and the cords CA, CB. Then ſince BC is perpendicular to AC, therefore (Prop. XVIII. Cor. 3.) the time of deſcending thro' the cord AC is equal to the time of falling thro' AB.

Draw CD parallel to AB, and DB parallel to CA, then is CD equal to AB. And by reaſon of the parallels, the angle DBC = angle BCA = a right angle. Then ſince DB is perp. to CB, therefore (Cor. 3. Prop. XVIII.) a body will deſcend thro' the inclined plane CB, in the ſame time that another falls thro' CD, or which is the ſame thing, thro' its equal AB.

Cor. 1. *Hence the times of deſcending thro' all the cords of a circle drawn from* A *or* B, *are equal among themſelves.* Fig. 11.

Cor. 2. *The velocity gained by falling thro' the cord* CB, *is as its length* CB.

For the velocity gained in falling thro' CB is the ſame as is gained by falling thro' EB; and that velocity is to the velocity gained by falling thro' AB, as $\surd$BE to $\surd$AB (by Cor. 1. Prop. XIII.); that is, as BC to BA. Therefore if the given velocity in falling down AB be repreſented by AB. The velocity gained in falling down CB will be repreſented by CB; and ſo that in any other cord, by its length.

PROP. XXIV.

If a pendulum vibrates in the ſmall arch of a circle; the time of one vibration, is to the time of a body's falling perpendicularly thro' half the length of the pendulum; as the circumference of a circle, to the diameter. 12.

If a pendulum ſuſpended by a thread, &c. be made to vibrate in any curve; it is the ſame thing as if it deſcended down a ſmooth poliſhed body made in the form of that curve. For the motions, velocities, and times of moving will be the ſame in both.

Let OD or OE be the pendulum vibrating in the arch ADC, whoſe radius is OD. Let OD be perp. to the horizon, and take the arch E*e* infinitely ſmall, and draw ABC, EFG, *efg*, perp. to OD; and draw the cord AD. About BD deſcribe the ſemicircle BGD. Draw *er* and G*s* perp. to EG.

Put $t =$ time of deſcending thro' the diameter 2OD, or thro' the cord AD. Then the velocities gained

Fig. 12. gained by falling thro' 2OD, and by the pendulum's deſcending thro' the arch AE, will be as $\sqrt{2OD}$ and $\sqrt{BF}$. And the ſpace deſcribed in the time t, after the fall thro' 2OD, is 4OD. But the times are as the ſpaces, divided by the velocities. Therefore, $\frac{4OD}{\sqrt{2OD}}$ or $2\sqrt{2OD} : t$ (time of its deſcription) : : $\frac{Ee}{\sqrt{BF}}$: time of deſcribing $Ee = \frac{t \times Ee}{2\sqrt{2OD \times BF}}$.

But by the ſimilar triangles OEF, Eer; and KGF, Ggs; we ſhall have $\frac{EF}{OD} \times Ee = er = Ff = Gs = \frac{FG}{KD} \times Gg$. Whence $Ee = \frac{OD \times FG}{KD \times EF} \times Gg$. Therefore the time of deſcribing $Ee =$

$$\frac{t \times OD \times FG \times Gg}{2KD \times EF \sqrt{BF \times 2OD}} = \frac{t \times OD \times \sqrt{BF \times FD} \times Gg}{2KD \sqrt{BF} \times \sqrt{DO + OF \times FD} \times \sqrt{2OD}} = \frac{t \times \sqrt{OD} \times Gg}{2KD \times \sqrt{DO + OF} \times \sqrt{2}} = \frac{t \times \sqrt{2OD}}{4KD \times \sqrt{DO + OF}} \times Gg. = \frac{t \times \sqrt{2OD}}{2BD \times \sqrt{2OD - DF}} \times Gg.$$

But DF, in its mean quantity for all the arches Gg, is nearly equal to DK. Therefore the time of deſcribing $Ee = \frac{t \times \sqrt{2OD}}{2BD \sqrt{2OD - DK}} \times Gg$. Whence the time of deſcribing the arch $AED = \frac{t \times \sqrt{2OD}}{2BD \sqrt{2OD - DK}} \times BGD$. And the time of deſcribing the whole arch ADC, or the time of one oſcillation is =

$t \times$

$\frac{t \times \sqrt{2OD}}{2BD\sqrt{2OD - DK}} \times 2BGD$. But when the arch ADC is exceeding ſmall, DK vaniſhes, and then the time of oſcillation in a very ſmall arch is $= \frac{t \times \sqrt{2OD}}{2BD\sqrt{2OD}} \times 2BGD = \frac{1}{2} t \times \frac{2BGD}{BD}$. But if t be the time of deſcending thro' 2OD, $\frac{1}{2} t$ is the time of deſcending thro' $\frac{1}{2}$ OD. And therefore BD the diameter, is to 2BGD the circumference; as the time of falling thro' half the length of the pendulum, to the time of one vibration. Fig. 12.

Cor. 1. *In a ſmall arch* AED, *the time of deſcending thro' the cord* AD, *is to the time of deſcending thro' the arch* AED; *as the diameter* BD, *to* $\frac{1}{4}$ *the circumference.*

For the time of deſcending thro' the arch AED $= t \times \frac{BGD}{2BD}$; therefore BD : $\frac{1}{2}$ BGD : : t : time in AED.

Cor. 2. *All the vibrations of the ſame pendulum, in arches not very large, are performed nearly in the ſame time.*

Cor. 3. *If* KD *be biſſected in* L, *and* T *be = time of vibration in a very ſmall arch. Then* $T + \frac{KL}{DO + OK} \times T$ *will be the time of vibration in any arch* ADC, *nearly.*

For we found the time of vibration in ADC $= \frac{t \times BGD}{BD} \times \sqrt{\frac{2OD}{2OD - DK}} = T \times \sqrt{\frac{2OD}{OD + OK}}$; and the three lines DO + OK, DO + OL, and DO + OD are in arithmetical progreſſion; but ſince KD is very ſmall, they are nearly in geometrical

Fig. 12. cal progreſſion; whence $\sqrt{\frac{2OD}{DO+OK}} = \frac{DO+OL}{DO+OK}$. Therefore the time of vibration $= T \times \frac{DO+OL}{DO+OK}$ $= T \times \frac{DO+OK+KL}{DO+OK} = T + T \times \frac{KL}{DO+OK}$.

Cor. 4. *Hence a falling body will deſcend thro' a ſpace of* 16 *feet, and* 1 *inch, in a ſecond of time.*

For by obſervation, a pendulum 39.13 inches long will ſwing ſeconds. And $t \times \frac{BGD}{BD} = 1$ ſecond, and $\frac{BD}{BGD} = t$, or $\frac{2}{3.1416}$ = time of falling thro' 2 × 39.13. Whence (Prop. XIII.) $\frac{4}{\overline{3.1416}^2} : 2 \times 39.13 :: 1^2 : \frac{39.13}{2} \times \overline{3.1416}^2 = 193.096$ inches = 16.09 feet.

PROP. XXV.

The lengths of two pendulums, deſcribing ſimilar arches, are as the ſquares of the times of vibration.

For (Prop. XXII.) the times of deſcending thro' two ſimilar curves, are as the ſquare roots of the lengths of the curves; that is, as the ſquare roots of the lengths of the pendulum, their centers being alike ſituated. Therefore the lengths of the pendulums are as the ſquares of the times of vibrating.

Cor. 1. *The times of vibration of pendulums in ſmall arches of circles, are as the ſquare roots of the lengths of the pendulums.*

For if the arches are ſimilar, the times of vibration are in that proportion. And (Prop. XXIV.

Cor. 2.) if the arches are ſmall, tho' not ſimilar, the vibrations will be the ſame as before. Fig.

Cor. 2. *The velocity of a pendulum at its loweſt point, is as the cord of the arch it deſcends thro'.*

For the velocity at the loweſt point is equal to the velocity gained in deſcending thro' the cord; for they are both of them the ſame as a body acquires by falling thro' their common altitude. And (Prop. XXIII. Cor. 2.) the velocity gained in falling thro' the cord, is as the length of the cord. Therefore the velocities of a pendulum in different arches, are in the ſame ratio.

PROP. XXVI.

Pendulums of the ſame length vibrate in the ſame time, whether they be heavier or lighter.

For let the two pendulums P, *p*, be of the ſame length; they will each of them fall thro' half the length of the pendulum in the ſame time. For (Cor. 2. Prop. V.) the velocity generated in any given time, is as the force directly and matter reciprocally. But in the two pendulums, the forces that generate their motions, are their weights, which are as their quantities of matter. Whence we have the velocity of P, to the velocity of *p*; as $\frac{P}{P}$ to $\frac{p}{p}$, or as 1 to 1; and therefore equal velocities are generated in the ſame time. Conſequently, equal ſpaces will be deſcribed in the ſame time, and therefore they will fall thro' half the length of one of them in an equal time. And therefore (Prop. XXIV.) their vibrations will be performed in the ſame time.

Cor. *Hence all bodies whether greater or leſſer, heavier or lighter, near the earth's ſurface will fall thro'*

Fig. *thro' equal ſpaces in equal times; abating the reſiſtance of the air.*

Becauſe they are as much retarded by their matter, as accelerated by their weight. The weight and the matter being exactly proportional to one another.

PROP. XXVII.

The lengths of pendulum's vibrating in the ſame time, in different places of the world, will be as the forces of gravity.

For (by Prop. V. Cor. 2.) the velocity generated in any time is as the force of gravity directly, and the quantity of matter reciprocally. And the matter being ſuppoſed the ſame in both pendulums, the velocity is as the force of gravity; and the ſpace paſſed thro' in a given time, will be as the velocity; that is, as the gravity. Therefore if any two ſpaces be deſcended thro' in any time, and two pendulums be made, whoſe lengths are double theſe ſpaces; theſe pendulums (by Prop. XXIV.) will vibrate in equal times; therefore the lengths of the pendulums, being as the ſpaces fallen thro' in equal times, will be as the forces of gravity.

Cor. 1. *The times wherein pendulums of the ſame length will vibrate, by different forces of gravity; are reciprocally as the ſquare roots of the forces.*

For (Cor. 2. Prop. V.) when the matter is given, the velocity generated is as the force × by the time. And (Prop. VI.) the ſpace deſcended thro' by any force, is as the force and ſquare of the time. Let theſe ſpaces be the lengths of the pendulums, then the lengths of the pendulums are as the forces and the ſquares of the times of falling thro' them. But (Prop. XXIV.) the times of falling thro' them are in a given ratio to the times of vibration;

whence the lengths of pendulums are as the forces and the ſquares of the times of vibration; therefore when the lengths are given, the forces will be reciprocally as the ſquares of the times; and the times of vibration reciprocally as the ſquare roots of the forces.

Cor. 2. *The lengths of pendulums in different places, are as the forces of gravity, and the ſquares of the times of vibration.*

This is proved under Cor. 2. Hence,

Cor. 3. *The times wherein pendulums of any length, perform their oſcillations; are as the ſquare roots of their lengths directly, and the ſquare roots of the gravitating forces reciprocally.*

Cor. 4. *The forces of gravity in different places, are as the lengths of pendulums directly, and the ſquares of the times of vibration reciprocally.*

PROP. XXVIII. *Prob.*

To find the length of a pendulum, that ſhall make any number of vibrations in a given time.

Reduce the given time into ſeconds, then ſay, as the ſquare of the number of vibrations given : to the ſquare of this number of ſeconds : : ſo is 39.13 : to the length of the pendulum ſought, in inches.

Ex. Suppoſe it makes 50 vibrations in a minute, here a minute is $=$ 60 ſeconds; then,

As 2500 (the ſquare of 50) : 3600 (the ſquare of 60) : : 39.13 : to the length $= \frac{3600 \times 39.13}{2500}$

$$= \frac{140868}{2500} = 56.34 \text{ inches, the length required.}$$

If

Fig. If it be required to find a pendulum that ſhall vibrate ſuch a number of times in a minute; you need only divide 140868, by the ſquare of the number of vibrations given, and the quotient will be the length of the pendulum.

This practice is deduced from Prop. XXV. for let p be the length of the pendulum, n the number of vibrations, t the time they are to be performed in. Then $39.13 : 1^2 :: p : \frac{p}{39.13} =$ ſquare of the time of one vibration, and $\sqrt{\frac{p}{39.13}} =$ time of one vibration; then if t be divided by $\sqrt{\frac{p}{39.13}}$ it will give n; that is, $t\sqrt{\frac{39.13}{p}} = n$, whence $tt \times 39.13 = nnp$, and $nn : tt :: 39.13 : p$.

If the pendulum is a thread with a little ball at it, then the diſtance between the point of ſuſpenſion and the center of the ball is eſteemed the length of the pendulum. But if the ball be large, ſay as the diſtance between the point of ſuſpenſion, and the center of the ball, is to the radius of the ball; ſo the radius of the ball to a third proportional. Set $\frac{2}{5}$ of this from the center of the ball downward, gives the center of oſcillation. Then the whole diſtance from the point of ſuſpenſion to this center of oſcillation, is the true length of the pendulum.

13. If the bob of the pendulum be not a whole ſphere, but a thin ſegment of a ſphere, as AB, as in moſt clocks; then to find the center of oſcillation, ſay as the diſtance between the point of ſuſpenſion, and the middle of the bob, is to half the breadth of the bob; ſo half the breadth of the bob, to a third proportional. Set one third of this length from the middle of the bob downwards, gives the center of oſcillation. Then the diſtance

between the centers of ſuſpenſion and oſcillation, is Fig. the exact length of the pendulum.

PROP. XXIX. *Prob.*

Having the length of a pendulum given; to find how many vibrations it ſhall make, in any given time.

Reduce the time given into ſeconds, and the pendulum's length into inches; then ſay, as the given length of the pendulum : to 39.13 : : ſo is the ſquare of the time given : to the ſquare of the number of vibrations, whoſe ſquare root is the number ſought.

Example. Suppoſe the length of the pendulum is 56.34 inches, to find how often it will vibrate in a minute.

1 minute = 60 ſeconds. Then, 56.34 (the length of the pendulum) : 39.13 : : 3600 (the ſquare of 60) : to the ſquare of the number of vibrations $= \frac{3600 \times 39.13}{56.34} = \frac{140868}{56.34} = 2500$, and $\sqrt{2500} = 50 =$ the number of vibrations ſought.

If the time given be a minute, you need only divide 140868 by the length, and extract the root of the quotient for the number of vibrations.

This is the reverſe of the laſt problem, therefore ſuppoſing as before in that problem, we have $tt \times 39.13 = nnp$; therefore $p : 39.13 : : tt : nn$.

They that would ſee a further account of the motions of bodies upon inclined planes, the vibrations of pendulums, and the motion of projectiles; may conſult my large book of Mechanics, where they will meet with full ſatisfaction.

SECT. III.

Of the Center of Gravity; the equilibrium of beams of timber; the directions and quantities of the forces neceſſary to ſuſtain them.

PROP. XXX.

A body cannot deſcend or fall downwards, except only when it is in ſuch a poſition, that by its motion, its center of gravity deſcends.

14. Let the body A ſtand upon the horizontal plane BK, and let C be its center of gravity; draw CD perpendicular to the plane BK. And let the body be ſuſpended at the point C, upon the perpendicular line CD. Then (def. 12.) it will remain unmoved upon the line CD. And as CD is perp. to the horizon, it has no inclination to move one way more than another, therefore it will move no way but remain at reſt. Take away the line CD, and let the body be ſupported by the line BC; if BC be fixed, the body will remain at reſt on the line CB. But if CB be movable about B, the body ſuſpended at C, will endeavour to move with its center of gravity downwards along the arch CE, about B as a center, towards N. And for the ſame reaſon the body will endeavour to fall the contrary way, moving about the point N; I ſay, this will be the caſe when CD is ſituated between B and N. But theſe two motions being contrary to one another, will hinder each other's effects; and the body will be ſuſtained without falling.

Again, let the body F be ſuſpended with its center of gravity I upon the perpendicular IH. As this line has no inclination to move to any ſide, it will therefore remain at reſt. Take away the line IH, and let the center of gravity I be ſuſpended on the line IG, then the body will endeavour to deſcend along the arch IK, for the higheſt point of the arch is in the perpendicular erected at G. For the ſame reaſon if the body be ſuſpended on the line OI, it will endeavour to deſcend towards K, about the center O; now as both theſe motions tend the ſame way, and there is nothing to oppoſe them; the body muſt fall towards K. In both theſe caſes it is plain, that when the center of gravity by its motion, deſcends, the body will fall; but if not, the body will be ſupported without falling. Fig. 14.

Cor. 1. *If a body ſtands upon a plane, if a line be drawn from the center of gravity perpendicular to the horizon; if this line fall within the baſe on which the body ſtands, it will be ſupported without falling. But if the perpendicular falls without the baſe, the body will fall.*

For when the perpendicular falls within the baſe, the body can be moved no manner of way, but the center of gravity will riſe. And when the perpendicular falls without the baſe, towards any ſide; if the body be moved towards that ſide, the center of gravity deſcends; and therefore the body will fall that way.

Cor. 2. *If a perpendicular drawn from the center of gravity perp. to the horizon, fall upon the extremity of the baſe; the body may ſtand, but the leaſt force whatever, will cauſe it to fall that way. And the nearer the perp. is to any ſide, the eaſier it will be made to fall, or is ſooner thruſt over. And the nearer the perp. is to the middle of the baſe, the firmer the body ſtands.*

Fig. 14. Cor. 3. *Hence if the center of gravity of a body be ſupported, the whole body is ſupported. And the place of the center of gravity muſt be deemed the place of the body; and is always in a line drawn perpendicular to the horizon, thro' the center of gravity.*

Cor. 4. *Hence all the natural actions of animals may be accounted for from the properties of the center of gravity.*

When a man endeavours to walk, he ſtretches out his hind leg, and bends the knee of his fore leg, by which means his body is thruſt forward, and the center of gravity of his body is moved forward beyond his feet; then to prevent his falling, he immediately takes up his hind foot, and places it forward beyond the center of gravity; then he thruſts himſelf forward, by his leg which now is the hindmoſt, till his center of gravity be beyond his fore foot, and then he ſets his hind foot forward again; and thus he continues walking as long as he pleaſes.

In ſtanding, a man having his feet cloſe together cannot ſtand ſo firmly, as when they are at ſome diſtance; for the greater the baſe, the firmer the body will ſtand; therefore a globe is eaſily moved upon a plane, and a needle cannot ſtand upon its point, any otherwiſe than by ſticking it into the plane.

When a man is ſeated in a chair, he cannot riſe till he thruſts his body forward, and draws his feet backward, till the center of gravity of his body be before his feet; or at leaſt upon them; and to prevent falling forward, he ſets one of his feet forward, and then he can ſtand, or ſtep forward as he pleaſes.

All other animals walk by the ſame rules; firſt ſetting one foot forward, that way the center of gravity leans, and then another.

In walking up hill, a man bends his body forward, that the center of gravity may lie forward of his feet; and by that means he prevents his falling backwards.

In carrying a burthen, a man always leans the contrary way that the burthen lies; ſo that the center of gravity of the whole of his body and the burthen, may fall upon his feet.

A fowl going over an obſtacle, thruſts his head forward, by that means moving the center of gravity of his whole body forwards; ſo that by ſetting one foot upon the obſtacle, he can the more eaſily get over it.

PROP. XXXI.

In any two bodies A, B, *the common center of gravity* C, *divides the line joining their centers, into two parts, which are reciprocally as the bodies.* AC : BC : : B : A. 15.

Let the line ACB be ſuppoſed an inflexible lever; and let the lever and bodies be ſuſpended on the point C. Then let the bodies be made to vibrate about the immovable point C; then will A and B deſcribe two arches of circles about the center C, and theſe arches will be as the velocities of the bodies, and theſe arches are alſo as the radii of the circles AC and BC. Therefore their velocities are as the radii. Whence, velocity of A : velocity of B : : AC : CB : : (by ſuppoſition) B : A. Therefore A × velocity of A = B × velocity of B. Whence (Prop. II.) the quantities of motion of the bodies A and B are equal, and (Ax. 9.) therefore they cannot move one another, but muſt remain at reſt; and conſequently (def. 12.) C is the center of gravity of A and B.

Fig. 15. Cor. 1. *The products of each body multiplied by its diſtance from the common center of gravity, are equal.* CA × A = CB × B.

Cor. 2. *If a weight be laid upon* C, *a point of the inflexible lever* AB, *which is ſupported at* A *and* B; *the preſſure at* A *to the preſſure at* B, *will be as* CB *to* CA.

For let the bodies A, B, be both placed in C; then (Cor. 3. Prop. XXX.) ſince it is the ſame thing whether the bodies be at A and B, or both of them at C, their center of gravity; therefore the preſſures at A and B will be the ſame in both caſes. But when they are at A and B, upon the lever ACB, their preſſures are A and B, being the ſame with the weights; therefore when they are both at C, the preſſures at A and B will ſtill be A and B. Therefore (Cor. 1.) ſince it is CA × A = CB × B; therefore CA : CB : : B : A : : preſſure at B : preſſure at A.

PROP. XXXII.

15. *If there be three or more bodies, and if a line be drawn from one body* E *to the center of gravity of the reſt* C. *Then the center of gravity of all the bodies divides the line* CD, *in two parts in* D, *which are reciprocally as the body* E *to the ſum of all the other bodies.* CD : DE : : E : A + B *&c.*

For ſuppoſe AB and CE to be two inflexible lines; and let the body W = A + B &c. and let W be placed in the center of gravity C. Then by the laſt Prop. CD : DE : : E : W or A + B &c.

Cor. *The body* E × DE *the diſtance from the common center of gravity, is equal to the ſum of the bodies* A + B *&c.* × *by* DC *the diſtance of their center from the common center of gravity.*

PROP. XXXIII.

If A, B, *be two bodies,* C *their center of gravity.* 16.
F *any point in the line* AB. *Then will* $FA \times A + FB \times B = FC \times \overline{A + B}$.

For (Cor. 1. Prop. XXXI.) $CA \times A = CB \times B$; that is, $\overline{FA - FC} \times A = \overline{FC - FB} \times B$; whence, by tranſpoſition $FA \times A + FB \times B = FC \times \overline{A + B}$.

Cor. *Hence the bodies* A *and* B *have the ſame force to turn the lever* AF *about the point* F, *as if they were both placed in* C *their center of gravity.*

PROP. XXXIV.

If ſeveral bodies A, B, E *&c. be placed on an* 17.
inflexible ſtreight lever; and if D *be their common center of gravity; and if* F *be any point in the line* AE, *then* $FA \times A + FB \times B + FE \times E$ *&c.* $= FD \times \overline{A + B + E}$ *&c.*

For if $A + B = W$, then $FA \times A + FB \times B + FE \times E = FC \times \overline{A + B} + FE \times E = FC \times W + FE \times E =$ (Prop. XXXIII.) $FD \times \overline{W + E} = FD \times \overline{A + B + E}$, in the three bodies A, B, E. And after the ſame manner, if there be four bodies, put $W = A + B + E$, and it will be proved the ſame way, that the ſum of all the products, $FA \times A + FB \times B$ &c. = diſtance of the common center of gravity × by all the four. And ſo on for more bodies.

Cor. *The ſame Prop. will hold good, when the bodies are not in the line* AF, *but any where in the perpendiculars paſſing thro' the points* A, B, E *&c.*

PROP. XXXV.

17. *If there be any number of bodies* A, B, E, *&c. either placed in the line* AF, *or any way in the perpendiculars paſſing thro'* A, B, E. *And if* D *be the center of gravity of all the bodies; and* F *be any point in the line* AF. *Then the diſtance of the center of gravity* FD $= \dfrac{FA \times A + FB \times B + FE \times E}{A + B + E}$.

For whether the bodies be in the points A, B, E, or in the perpendiculars, it will be (by Prop. XXXIV. and Cor.) that $FA \times A + FB \times B + FE \times E = FD \times \overline{A + B + E}$. Whence $FD = \dfrac{A \times FA + B \times FB + E \times FE}{A + B + E} =$ ſum of all the products of each body multiplied by its diſtance, divided by the ſum of the bodies.

Cor. 1. *If a ſingle body only was placed on the lever* AF; *then the diſtance of the center of gravity of that body, is equal to the ſum of the products of all the particles of the body, each multiplied by its diſtance from a given point* F, *and divided by the body.*

For if A, B, E &c. are ſeveral particles of the body, then A + B + E &c. = the body; and

$$FD = \frac{A \times FA + B \times FB + E \times FE}{\text{body}}.$$

Cor. 2. *If there be ſeveral bodies* A, B, E, *&c. placed upon the lever* AF. *They act with the ſame force in turning the lever about any given point* F, *as if they were all placed in* D *the common center of gravity of all the bodies.*

SCHOLIUM.

If any of the bodies be placed on the contrary ſide of F, *their reſpective products will be negative.*

For they act the contrary way in turning the lever about. Fig. 17.

PROP. XXXVI.

18. *If ſeveral bodies* A, B, E, G, H, *be placed on the lever* AH, *and* F *be the center of gravity of all the weights. Then* $FA \times A + FB \times B + FE \times E = FG \times G + FH \times H$.

For let the lever be ſuſpended on the point F, then the two ends will be in equilibrio, as F is the center of gravity. Let D be the center of gravity of A, B, E; and I the center of gravity of G, H. Then (Cor. Prop. XXXIII.) it is the ſame thing whether the bodies on one ſide be placed at A, B, E, or all of them in the point D. And whether thoſe at the other end be placed at G, H; or all of them at I. But ſince F is the center of gravity, $DF \times \overline{A + B + E} = FI \times \overline{G + H}$, and therefore $A \times AF + B \times BF + E \times EF = G \times GF + H \times HF$ (by Prop. XXXIV.)

Cor. 1. *If ſeveral bodies* A, B, E, G, H, *be placed on an inflexible lever, and if* $A \times FA + B \times FB + E \times FE = G \times FG + H \times FH$. *Then* F *is the center of gravity of all the bodies.*

For no other point will anſwer the equation.

Cor. 2. *If ſeveral bodies* A, B, E, G, H, *be placed upon a lever* AH, *or ſuſpended at theſe points by ropes; and if* $A \times FA + B \times FB + E \times FE = G \times FG + H \times FH$; *they will be in equilibrio upon the point* F.

This appears by Def. 12, and F is the center of gravity.

PROP.

Fig.

PROP. XXXVII.

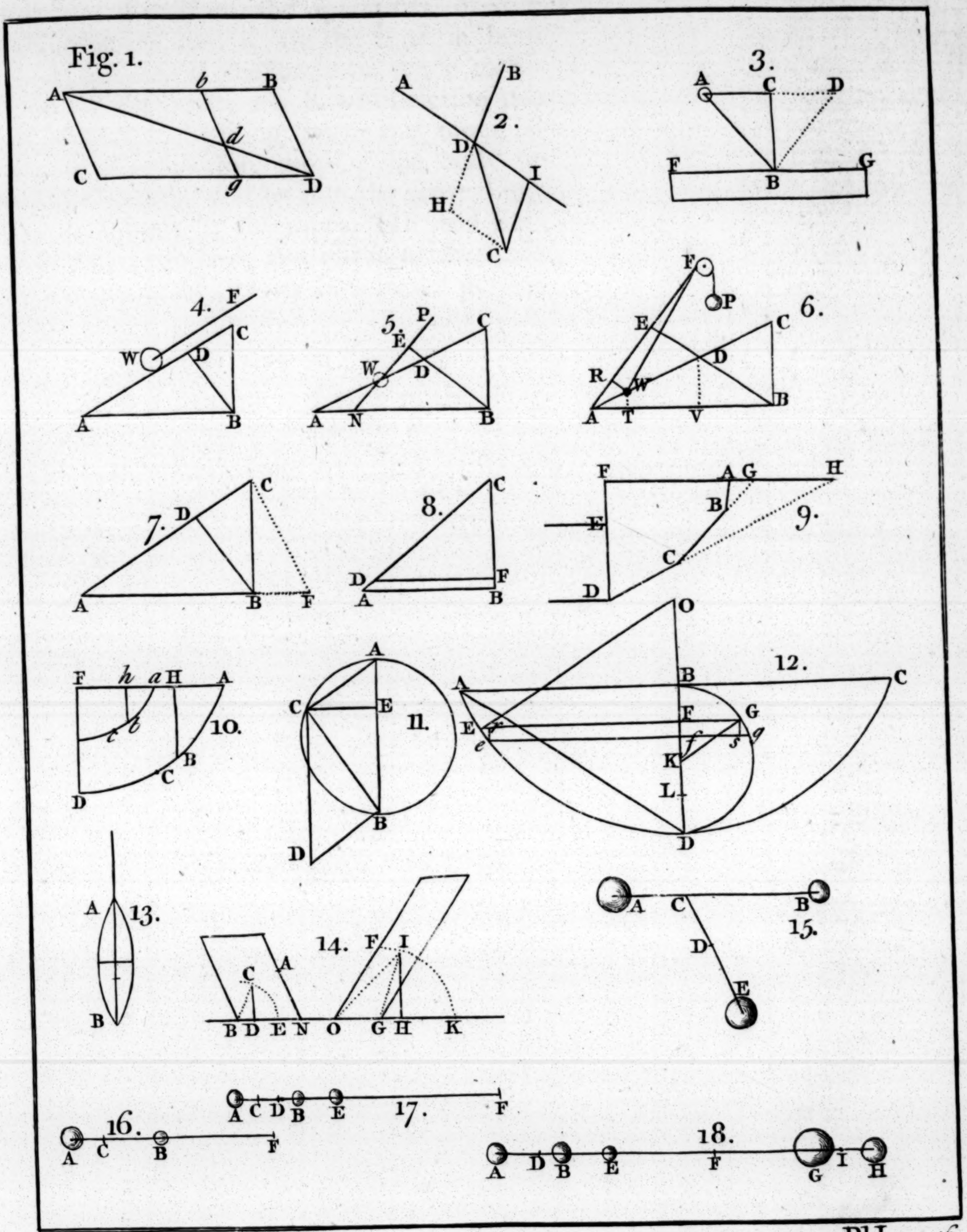
19. *If a heavy body* AB, *ſuſpended by two ropes* AC, BD, *remains at reſt*; *a right line perpendicular to the horizon, paſſing thro' the interſection* F *of the ropes*; *will alſo paſs thro' the center of gravity* G, *of the body.*

If AC and BD be produced to F where they interſect; then (ax. 12.) it is the ſame thing whether the force acting in direction AC be applied to C or F; and whether the force acting in direction BD be applied to the point D or F. Suppoſe then that they both act at F, and then it is the ſame thing, as if the body was ſuſpended at F by the two ſtrings AF, BF. And ſince the body is at reſt, therefore (Ax. 7.) the body, that is, the center of gravity G, is at the loweſt place it can get; and therefore is in the plumb line FG. For if the body be made to vibrate, the center of gravity G will deſcribe an arch of a circle, of which G (being in the perp. FG) is the loweſt point.

Cor. 1. *Hence if* GN *be drawn parallel to* AC; *then the weight of the body, the forces acting at* C *and* D, *are reſpectively as* FG, GN, *and* FN.
This is evident by Prop. VIII.

Cor. 2. *If a heavy body* AB, *be ſupported by two planes*, IKL, *and* EHG, *at* H and K; *and* HF *and* KF *be drawn perpendicular to theſe planes*; *and if* FG *be drawn from the interſection* F, *perp. to the horizon, it will paſs thro' the center of gravity G, of the body.*

For ſince the body is ſuſtained by theſe planes, therefore the planes re-act againſt the body (by Prop. IX.), in the directions HF, KF pependicular to theſe planes. Therefore it is the ſame thing as



Mechanics

Pl. I. *pa. 46.*

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as if the body was ſuſtained by the two ropes HF, KF. For the directions and quantities of the forces, acting at H and K are the ſame in both caſes. And further, if the body be made to vibrate round F, the points H, K will deſcribe two arches of circles, coinciding with the touching planes at H, K; therefore moving the body up and down the planes, will be juſt the ſame thing, as making it vibrate in the ropes, HF and KF; and conſequently, the body can reſt in neither caſe, but when the center of gravity G is in the perpendicular FG. Fig. 19.

SCHOLIUM.

If any body ſhould deny the truth of this Prop. or its corollaries, againſt the cleareſt force of demonſtration. It lies upon them to ſhew where the demonſtration fails, or what ſtep or ſteps thereof do not hold good, or are not truly deduced from the foregoing. If they cannot do this, what other reaſons they may aſſign, can ſignify nothing at all to the purpoſe. And if ſuch perſon, ignorant of the laws of nature, and the reſolution of forces, would object againſt this practice, of ſubſtituting planes perpendicular to the lines or cords ſuſtaining any weights, inſtead of theſe cords. Let him firſt read Sir *J. Newton*'s *Principia*, Cor. 2. to the laws of nature, where he will ſee this practice exemplified, and then make his objections.

And for the ſake of ſuch perſons as underſtand not how to apply the method of compoſition and reſolution of forces, I ſhall add a few problems to prevent their being miſled by the raſh judgment of ſome people, who having brought out falſe ſolutions to ſome problems by their own ill management, condemn the method as erroneous; when the fault really lies in their own ignorance, and not at all in the method itſelf.

PROP.

Fig.

PROP. XXXVIII. *Prob.*

20. *To determine the poſition of a beam* CD, *movable about the end* C, *and ſuſtained by a given weight* Q, *hanging at a rope* QAD, *which goes over a pulley at* A *and is fixed to the other end* D.

Draw AF, CK parallel to the horizon, FDE perp. to it, and KD perp. to CD; and let G be the center of gravity, *w* = weight of the beam. Then if the beam was to lie horizontally (Prop. XXXI. Cor. 2.) it would be GC : GD : : preſſure at D : preſſure at C; and GC : CD : : preſſure at D : *w*; whence the preſſure at D $= \frac{GC}{CD} w$, horizontally. And (Prop. XIV.) CD : CE : : $\frac{GC}{CD} w$; $\frac{CE \times CG}{CD^2} w =$ preſſure in direction DK. Produce AD to O, and draw OI parallel to CD. Then the beam is ſuſtained by three forces in directions OD, DI and IO; and DI : DO : : S.IOD or ODC or ADC : rad; whence S.ADC : rad : : $\frac{CE \times CG}{CD^2} \times w$: force DO or Q $= \frac{\text{rad} \times CE \times CG}{S.ADC \times CD^2} w$. Therefore *w* : Q : : S.ADC : rad $\times \frac{CE \times CG}{CD^2}$: : S.ADC : $\frac{CG}{CD} \times$ S.FDC, becauſe $\frac{\text{rad} \times CE}{CD} =$ S.EDC or FDC.

PROP. XXXIX. *Prob.*

21. *Let the beam* ED, *be ſuſtained by the weights* P, Q, *by means of the ropes* DCP, EAQ, *going over the pulleys* C, A, *in the horizontal line* AC. *To find the poſition of the beam; having the weights* P, Q, *given.*

1 *Way.*

Let G be the center of gravity of the beam. Thro' D, E, draw HDS, FER perpendicular to AC.

Fig. 21.

AC. Then (Cor. 2. Prop. VIII.) S.EDS : P the tenſion of the thread CD : : S.CDS or CDH : $\frac{S.CDH}{S.EDS} \times P$ the tenſion of DE. Alſo, S.DER or EDS : Q : : S.AER or AEF : $\frac{S.AEF}{S.EDS} \times Q$ the tenſion of DE in a contrary direction. Then as the beam is in equilibrio, theſe forces or tenſions balance one another; therefore $\frac{S.CDH}{S.EDS} \times P = \frac{S.AEF}{S.EDS} \times Q$. Then P : Q : : S.AEF : S.CDH; which may be otherwiſe expreſſed, for AE : rad : : AF : S.AEF $= \frac{AF}{AE} \times$ rad; and DC : rad : : HC : $\frac{HC}{DC} \times$ rad $=$ S.HDC. Whence P : Q : : $\frac{AF}{AE} : \frac{CH}{CD} : : \frac{AF}{HC} : \frac{AE}{DC}$.

2 *Way*.

Let R, S be the perpendicular preſſures of the ends E, D. $w =$ weight of the beam. Then (Cor. 2. Prop. XXXI.) $R = \frac{DG}{ED} w$, and $S = \frac{EG}{ED} w$. And (Cor. 2. Prop. VIII.) S.AED : R or $\frac{DG}{ED} w$: : S.AEF : tenſion of DE $= \frac{S.AEF \times DG}{S.AED \times ED} w$. And S.CDE : S or $\frac{EG}{ED} w$: : S.CDH : contrary tenſion of DE $= \frac{S.CDH \times EG}{S.CDE \times ED} w$, and theſe two forces of DE being equal, we have $\frac{S.AEF \times DG}{S.AED \times ED} w =$

Fig. 21. $\frac{S.CDH \times EG}{S.CDE \times ED}w$, and $\frac{S.AEF \times DG}{S.AED} = \frac{S.CDH \times EG}{S.CDE}$.

Whence EG : DG : : $\frac{S.AEF}{S.AED} : \frac{S.CDH}{S.CDE}$: : S.AEF × S.CDE : S.CDH × S.AED.

3 *Way.*

S.CDE : S.EDS : : S : P $= \frac{S.EDS}{S.CDE} \times S$, and S.AED : S.RED : : R : $\frac{S.RED}{S.AED}R = Q$. Then P : Q : : $\frac{S.EDS}{S.CDE} \times S : \frac{S.RED}{S.AED} \times R$: : S.AED × S : S.CDE × R : : (laſt method) S.AED × $\frac{EG}{ED}w$: S.CDE × $\frac{DG}{ED}w$: : S.AED × EG : S.CDE × DG. And S.AED : S.CDE : : P × DG : Q × EG.

4 *Way.*

Draw C*m*, F*n* parallel to DE, and FE, HD perp. to the horizon. Then by the reſolution of forces, CD : D*m* : : P : $\frac{Dm}{CD}P$ = perpendicular force at D; and *n*E : EF : : Q : $\frac{EF}{nE}Q$ = perpendicular force at E. Therefore EG : GD : : $\frac{Dm}{CD}P : \frac{EF}{nE}Q$: : $\frac{S.CDE}{S.CmD} \times P : \frac{S.FnE}{S.nFE} \times Q$: : S.CDE × P : S.AED × Q. For S.C*m*D = S.*m*DE = S.FED = S.*n*FE. That is, EG : GD : : S.CDE × P : S.AED × Q. As in the third way.

5 *Way.*

Let R, S be the perpendicular weights of the ends E, D; or which is the ſame, the tenſions of the perpendicular ropes, FE, HD. By the reſolution of forces, if Cm, Fn be parallel to DE. The force FE or R is equivalent to En, Fn; and the force Dm or S, to DC, mC; therefore EF : Fn : : R : $\frac{Fn}{FE}$R = force at E in direction Fn. And Dm : mC : : S : $\frac{mC}{mD}$S = force at D in direction mC. But the beam being in equilibrio, theſe two oppoſite forces muſt be equal; therefore $\frac{Fn}{FE}R = \frac{mC}{mD}S$. Whence $R : S :: \frac{mC}{mD} : \frac{Fn}{FE} :: \frac{S.CDH}{S.CDE} : \frac{S.AEF}{S.AED} ::$ S.AED × S.CDH : S.AEF × S.CDE. But (Cor. 2. Prop. XXXI.) R : S : : DG : EG. Whence DG : EG : : S.AED × S.CDH : S.AEF × S.CDE; the ſame as by the 2d method. And the ſame thing likewiſe follows from the 1ſt and 4th method together. Fig. 21.

From theſe ſeveral ways of proceeding, it is evident, that which ever way we take, the proceſs if rightly managed always brings us to the ſame concluſion; and it comes to the ſame thing which way we uſe, ſo that we proceed in a proper manner. And this among other things, ſhews the great uſe and extent of that noble theory of the compoſition and reſolution of forces.

What is calculated above is concerning the angles, or the poſition of the ſeveral lines to one another, depending on the ſeveral forces. Then in regard to the weight of the beam, put it = w, then DC : Dm : : P : $\frac{Dm}{DC}$P = S, and En : EF : : Q : $\frac{EF}{En}$Q = R. And $w = R + S = \frac{Dm}{DC}P + \frac{EF}{En}Q$, an equation ſhewing the relation of the weights to one another.

Fig. 21.

6 Way, by the center of gravity.

Produce AE, CD to B, and from B draw BGO thro' the center of gravity; which (by XXXVII.) will be perp. to AC, and therefore parallel to EF, DH. Then the angle EBG = AEF, and DBG = CDH. Then EB : BD : : S.BDE or CDE : S.BED or AED, and (Trigonom. B. II. Prop. V. Cor. 1.) EG : GD : : EB × S.EBG : BD × S.DBG : : EB × S.AEF : BD × S.CDH : : S.CDE × S.AEF : S.AED × S.CDH; the ſame as by the 2d way. Whence all the reſt will be had as before.

Cor. *It will be exactly the ſame thing, whether the weights* P, Q, *remain, or the ſtrings* AE, CD, *be fixed in that poſition to two tacks, any way in theſe lines. And if a beam* ED, *hang upon two tacks* A, C, *by ropes fixed there; it makes no difference, if you put two pulleys inſtead of the tacks, for the ropes to go over, and then hang on the weights* Q, P, *equal to the tenſions of the ſtrings* AE, CD.

For in both caſes, the forces or the tenſions of the ſtrings, and their directions, remain the ſame. And there is nothing elſe to make a difference in the ſituation of the beam.

SCHOLIUM.

Every one that knows any thing of mechanical principles will eaſily underſtand, that if any forces, which keep a body at reſt, be reſolved into others, to have the ſame effect; the contrary forces, or thoſe directly oppoſite, muſt act againſt a ſingle point; or elſe the equilibrium will be deſtroyed. And therefore in the preſent Prop. ſuppoſe any one ſhould divide the forces CD, AE, into the two HD, DY, and FE, EX, one perpendicular, the other parallel to the horizon. The forces HD, EF, will indeed balance the force of gravity at D and E, to which

Fig. 21.

which they are directly oppoſite. And therefore the beam will remain unmoved by theſe. But the equal forces DY, EX, being parallel, never meet in a point; but acting obliquely on the beam, one of them drawing at D in direction DY, and the other at E in direction EX, the effect will be, that they will turn the beam ED about the center of gravity G. Therefore to prevent this, the forces DY, EX, muſt be ſubdivided; that is, they muſt be reſolved into others, one whereof is perp. to the horizon, the other parallel to ED. Then gravity will balance theſe perp. to the horizon, and the others, being equal and oppoſite, acting in the line EGD, act equally againſt any of the points D, G, or E. And ſo the beam will remain at reſt. But this is much better done at once at the firſt, by dividing DC, EF, each into two forces, one perp. to the horizon, the other parallel to the beam ED. And then the oppoſite forces will exactly balance one another, and the beam remain unmoved.

PROP. XL. *Prob.*

22. *To find the poſition of the beam* ED, *hanging by the rope* EBD, *whoſe ends are faſtened at* E *and* D, *and goes over a pulley fixed at* B.

Let G be the center of gravity of the beam, then (Prop. XXXVII.) BG will be perp. to the horizon. Then as the cord runs freely about the pulley B; therefore (Ax. 13.) the tenſion of the parts of the rope EB, BD are equal to one another, ſuppoſe = T. By the reſolution of forces, the force EB is equivalent to EG, GB; and DB to DG, GB. Therefore BE : EG : : T : $\frac{EG}{EB}$T = force in direction EG. And BD : DG : : T : $\frac{DG}{BD}$T = force in

Fig. 22. direction DG, which is equal and oppoſite to that in EG; therefore $\frac{EG}{EB}T = \frac{DG}{DB}T$. Whence EG : EB : : DG : DB. And therefore BG biſects the angle EBD.

Cor. *Hence* ED : *ſtring* EBD : : EG : EB *the part* EB *of the ſtring* : : *and ſo* GD : DB *the part* DB *of the ſtring.*

SCHOLIUM.

If GD be leſs than GE, then the center of gravity G, will be loweſt, when the beam hangs perpendicular with the end D downward. And in many caſes it will be higheſt, when it hangs perpendicular, with the end E downward.

PROP. XLI. *Prob.*

23. *There is a beam* BC *hanging by a pin at* C, *and lying upon the wall* BE; *to find the forces or preſſures at the points* B, *and* C, *and their directions.*

Produce BC to K, ſo that CK may be equal to CB. Draw BA parallel, and CL perpendicular, to the horizon; and draw BL, CN, KI perp. to BCK. Thro' the center of gravity G, draw GF parallel to CL. By Prop. XIV. if a body lies upon an inclined plane, as BC; its weight, its inclination down the plane, and preſſure againſt it, are as BC, CA and AB; that is, as CL, CB and BL. Therefore if CL repreſent the weight of the body, CB will be the force urging it down the plane, and BL the total preſſure againſt the plane. And ſince GF is parallel to CL, BL is divided in F, in the ſame ratio, as BC is divided in G. And therefore (Cor. 2. Prop. XXXI.) BF will be the part of the preſſure acting at C, and FL the part acting at B. Make CN equal to BF, and compleat the parallelogram CNIK, and draw CI. Then Fig. ſince BC or CK is the force in direction CK, and 23. CN the force in direction CN; then by compoſition, CI will be the ſingle force by which C is ſuſtained, and CI its direction. But the triangles CKI, CBF are ſimilar and equal, and CI = CF, and in the ſame right line; therefore CF is the quantity and direction of the force acting at C to ſuſtain it. Therefore the weight of the body, the preſſure at B, and the force at C; are reſpectively as CL, FL, and CF.

Cor. 1. *Produce* FG *to interſect* CN *in* H; *then the weight of the body, the preſſure at* B, *and the force acting at* C; *are reſpectively as* HF, HC, *and* CF.

For in the parallelogram CLFH, HF = CL, and HC = FL.

Cor. 2. *If the beam was ſupported by a pin at* B, *and laid upon the wall* AC; *the like conſtruction muſt be made at* B, *as has been done at* C, *and then the forces and directions will be had.*

Cor. 3. *If a line perp. to the horizon be drawn thro'* F, *where the direction of the forces* CF, *and* BF *meet; it paſſes thro'* G *the center of gravity of the beam.*

Cor. 4. *It is all one whether the beam is ſuſtained by the pin* C *and the wall* BE, *or by two ropes* CI, BP *acting in the directions* FC, FB, *and with the forces* CF, FL.

SCHOLIUM.

The proportions and directions of the forces here found, are the ſame as in Prop. LXIV. of my large book of Mechanics, and obtained here by a different method. The principles here uſed are indiſputable; and the principle made uſe of in that

Fig. LXIV. Prop. is here demonſtrated in the third Cor.
23. ſo that the reader may depend upon the truth of them all.

PROP. XLII. *Prob.*

24. BC *is a heavy beam ſupported upon two poſts* KB, LC; *to find the poſition of the poſts, that the beam may reſt in equilibrio.*

Let G be the center of gravity; draw BA parallel to the horizon, and BF, GD, CAN perpendicular to it. Then (Prop. XXXI. Cor. 2.) if BC be the weight of the body, CG will be the part of the weight acting at B, and BG the weight at C. Therefore make CN = BG, and BF = GC; and from N and F, draw NI, FK, parallel to BC; and make NI = FK, of any length, and lying contrary ways. Then draw IC and KB, which will be the poſition of the poſts required.

For BF is the weight upon B; and CN, that upon C, which forces being in direction of the lines BF, CN, the beam will remain at reſt by theſe forces. And the forces NI, FK, in direction BC, being equal and contrary, will alſo keep the beam in equilibrio, therefore the forces KB, IC, compounded of the others, will alſo keep the beam in equilibrio, acting in the directions KB, IC, or MB, LC.

Cor. 1. *Hence if* DG *be produced, it will paſs thro' the interſection* H, *of the lines* LC, MB.

For the triangles INC, CGH are ſimilar; therefore IN : NC : : CG : GH, the interſection with CL. Alſo the triangles KFB, BGH are ſimilar; therefore KF : BG : : BF : GH the interſection with MB, which muſt needs be the ſame as the other, ſince the three firſt terms of the proportion are the ſame; for KF = NI, BG = NC, and BF = CG.

Cor. 2. *If a line be drawn thro' the center of gravity* G, *of the beam, perpendicular to the horizon; and from any point* H *in that line, (above or below* G), *the lines* HBK *and* HCM *be drawn; then the props* BM *and* CL *will ſuſtain the beam in equilibrio.* Fig. 24.

Cor. 3. *If* GO *be drawn parallel to* HC; *then the weight of the beam, the preſſure at* C, *and thruſt or preſſure at* B; *are reſpectively as* HG, OG, *and* HO, *and in theſe directions.*

Cor. 4. *It is all one for maintaining the equilibrium, whether the beam* BC *be ſupported by two poſts or props* MB, LC; *or by two ropes* BH, CH; *or by two planes perpendicular to theſe ropes at* B *and* C.

For in all theſe caſes the forces and directions are the ſame; and there is nothing elſe concerned, but the forces and directions.

SCHOLIUM.

It does not always happen that the center of gravity is at the loweſt place it can get, to make an equilibrium. For here if the beam BC be ſupported by the poſts MB, LC; the center of gravity is at the higheſt it can get; and being in that poſition, it has no inclination to move one way more than another, and therefore it is as truly in equilibrio, as if it was at the loweſt point. It is true, any the leaſt force will deſtroy that equilibrium, and then if the beam and poſts be movable about the angles M, B, C, L, which is all along ſuppoſed, the beam will deſcend till it is below the points M, L, and gain ſuch a poſition as deſcribed in Prop. XXXIX. and its Cor. ſuppoſing the ropes fixed at A, C (fig. 21.); and then G will be at the loweſt point, and will come to an equilibrium again. In planes, the center of gravity G may be either at its higheſt or loweſt point. And there are caſes, when the

Fig. 24. the center of gravity is neither at its higheſt nor loweſt, as may happen in the caſe of Prop. XL. ſo that they who contend, that in caſe of an equilibrium, the center of gravity muſt *always* be at the loweſt place, are much miſtaken, and know little about the principles of mechanics, or the nature of things.

Thoſe that want to ſee more variety about the motion of bodies, on inclined planes; the preſſure, and direction of the preſſure of beams of timber; centers of gravity, and alſo the centers of oſcillation and percuſſion, &c. may conſult my large book of Mechanics.

SECT. IV.

The MECHANICAL POWERS; *the Strength and Streſs of Timber.*

PROP. XLIII.

IN a balance, where the arms are of equal length; if two equal weights be ſuſpended at the ends, they will be in equilibrio.

The balance is a ſtreight inflexible rod or beam, turning about a fixed point or axle in the middle of it; to be loaded at each end with weights ſuſpended there.

Let AB be the beam or lever, C the middle point or center of motion; D, E the weights hanging at the ends A and B. Then let the beam and the weights, or the whole machine, be ſuſpended at C. And ſuppoſe the beam and the weights be turned about upon the center C; then the points A, B being equidiſtant from C will deſcribe equal arches, and therefore the velocities will be equal, and if the bodies D and E be equal, then the motion of D will be equal to the motion of E, as their quantities of matter and velocities are equal; and conſequently, if the beam and weights are ſet at reſt, neither of them can move the other, but they will remain in equilibrio. 25.

Cor. *If one weight be greater than the other; that weight and ſcale will deſcend, and raiſe the other.*

SCHOLIUM.

The uſe of the balance, or a common pair of ſcales, is to compare the weights of different bodies;

Fig. 25. dies; for any body whoſe weight is required, being put into one ſcale, and balanced by known weights put into the other ſcale, theſe weights will ſhew the weight of the body. To have a pair of ſcales perfect, they muſt have theſe properties. 1. The points of ſuſpenſion of the ſcales, and the center of motion of the beam, A, C, B, muſt be in a right line. 2. The arms AC, BC, muſt be of equal length from the center. 3. That the center of gravity be in the center of motion C. 4. That there be as little friction as poſſible. 5. That they be in equilibrio, when empty.

If the center of gravity of the beam be above the center of motion, and the ſcales be in equilibrio, if they be put a little out of that poſition, by putting down one end of the beam, that end will continually deſcend, till the motion of the beam is ſtopt by the handle H. For by that motion, the center of gravity is continually deſcending, according to the nature of it. But if the center of gravity of the beam be below the center of motion; if one end of the beam be put down a little, to deſtroy the equilibrium; it will return back and vibrate up and down. For by this motion the center of gravity is endeavouring to deſcend.

PROP. XLIV. *Prob.*

25. *To make a falſe balance; or one which is in equilibrio when empty, and alſo in equilibrio, when loaded with unequal weights.*

Make ſuch a balance as deſcribed in the laſt Prop. only inſtead of making the center of motion in the middle at C, make it nearer one end, as at F. And make the weight of the ſcales ſuch, that they may be in equilibrio upon the center F. Then if two weights D, E, be made to be in equilibrio in the two ſcales; theſe weights will be unequal, for they will be reciprocally as the lengths of the arms AF, BF. That is, AF : BF : : E : D. Fig. 25.

For (Prop. XXXI. Cor. 1.) ſince F is the center of gravity of D and E, ſuppoſing them to act at A and B; therefore FA × D = FB × E. And FA : FB : : E : D. But AF is greater than FB, therefore E is greater than D.

Cor. 1. *Hence to diſcover a falſe balance, make the weights in the two ſcales to be in equilibrio; then change the weights to the contrary ſcales. And if they be not in equilibrio, the balance is falſe.*

Cor. 2. *Hence alſo to prove a pair of good ſcales, they muſt be in equilibrio when empty, and likewiſe in equilibrio with two weights. Then if the weights be changed to the contrary ſcales, the equilibrium will ſtill remain, if the ſcales are good.*

Cor. 3. *From hence alſo may be known what is gained or loſt, by changing the weights, in a falſe balance.*

Take any weight as 1 pound, to be put into one ſcale and balanced by any ſort of goods in the other. Since AF × D = BF × E; let the weight D be 1, then $E = \frac{AF}{BF}$ the weight of the goods in the ſcale E. Then changing the ſcales, let the weight E be 1; then $D = \frac{BF}{AF}$ the weight of the goods in the ſcale D. Then $\frac{AF}{BF} + \frac{BF}{AF} =$ whole weight of the goods, and $\frac{AF}{BF} + \frac{BF}{AF} - 2 =$ gain to the buyer in 2 lb. &c. Therefore $\frac{AF^2 + BF^2 - 2AF \times BF}{AF \times BF} =$ gain in 2 lb. $= \frac{\overline{AF - BF}^2}{AF \times BF}$; and $\frac{\overline{AF - BF}^2}{2AF \times BF} =$ gain in 1 lb.

There-

Fig. 25. Therefore if *w* is any weight to be bought; the gain to the buyer, for the weight *w*, by changing the ſcales, will be $\frac{\overline{\text{AF}-\text{BF}}^2}{2\text{AF}\times\text{BF}}w$. For example, if AF be 16, and BF 15; then the gain will be $\frac{\overline{16-15}^2}{2\times16\times15}w = \frac{1}{480}w$.

SCHOLIUM.

In demonſtrating the properties of the mechanical powers; ſince the weight is commonly ſome large body whoſe weight is to be overcome or balanced; therefore the power which acts againſt it, will be moſt fitly repreſented by another weight; and thus the power and weight being of the ſame kind, may moſt properly and naturally be compared together. For ſuch a weight may repreſent any power, as the ſtrength of a man's hand, the force of water or wind, &c. And this weight repreſenting the power, being ſuſpended by a rope, may hang perpendicular where the power is to act perpendicular to the horizon; or may go over a pulley, when it acts obliquely.

PROP. XLV.

26. *If the power and weight be in equilibrio upon any*
27. *lever, and act in lines perpendicular to the lever;*
28. *then the power* P *is to the weight* W; *as the diſ-*
29. *tance of the weight from the ſupport* C, *is to the diſtance of the power from the ſupport.*

There are four ſorts of levers. 1. When the ſupport is between the weight and the power. 2. When the weight is between the power and the ſupport. 3. When the power is between the weight and the ſupport. 4. When the lever is not ſtreight but crooked.

A lever is any inflexible rod or beam, of wood Fig.
or metal, made uſe of to move bodies. The ſup- 26.
port is the prop it reſts on, in moving or ſuſtaining 27.
any heavy body, and this is the ſame as the center 28.
of motion. 29.

Let the power P act at A by means of a rope; then as C is the prop or center of motion, if the lever be made to move about the center C, the arches deſcribed by A and W; that is, the velocities of A and W will be as the radii CA and CW. But the velocity of P is the ſame as that of the point A. Therefore velocity of P : velocity of W : : CA : CW : : (by ſuppoſition) W : P; therefore P × velocity of P = W × velocity of W. Conſequently their motions are equal, and they cannot move one another, but muſt remain in equilibrio. And if they be in equilibrio, they muſt have this proportion aſſigned.

Cor. 1. *If a power* P *act obliquely againſt the lever* WA; *the power and weight will be in equilibrio, when the power* P *is to the weight* W; *as the diſtance of the weight* CW, *to* CD *the perpendicular, drawn from the ſupport to the line of direction of the power.* 30.

For in this caſe WCD becomes a bended lever, and the power P acts perpendicular to CD at D; and (Ax. 12.) it is all one whether the power acts at D or A, in the line of direction AD. And hence,

Cor. 2. *If any force be applied to a lever* ACW *at* A, *its power to turn it about the center of motion* C, *is as the ſine of the angle of application* CAD.

For if CA be given, CD is as the ſine of CAD.

Cor. 3. *In a ſtreight lever, of theſe three, the power, the weight, and the preſſure upon the ſupport; the middlemoſt is equal to the ſum of the other two.*

For the middle one acts againſt both the others and ſupports them.

Fig. 30. Cor. 4. *From the foregoing properties of the lever, the effects of ſeveral ſorts of ſimple machines may be explained; and likewiſe the manner of lifting, carrying, drawing of heavy bodies, and ſuch like.*

26. For example, if a given weight W is to be raiſed by a ſmall power applied at A, the end of the lever AW. If we divide WA in C, ſo that it be as CA : CW : : as the weight W : to the power P; then fixing a prop or ſupport at C or rather a little nearer W; then the power P with a ſmall addition, will raiſe the weight W.

27. Again, if two men be to carry a weight W, upon the lever CA. The weight the man at A carries, is to the weight the man at C carries as CW, to AW. And therefore the lever or beam CA ought to be divided in that proportion at W, the place where the weight is to lie, according to the ſtrength of the men that carry it.

31. Likewiſe if two horſes be to draw at the ſwing-tree AB, by the ropes AF, BG; and the ſwing-tree draws a carriage &c. by the rope CD; then the force acting at A will be to the force acting at B, as BC to AC. And therefore BC ought to be to AC, as the ſtrength of the horſe at F, to the ſtrength of the horſe at G; the weaker horſe having the longer end. But it is proper to make the croſs bar AB crooked at C; that when the ſtronger horſe pulls his end more forward, he may pull obliquely, and at a leſs diſtance from the center; whilſt the weaker horſe pulls at right angles to his end, and at a greater diſtance.

Again, ſuch inſtruments as crows and handſpikes are levers of the firſt kind, and are very uſeful and handy for raiſing a great weight to a ſmall hight. Alſo ſciſſars, pinchers, ſnuffers, are double levers of the firſt kind, where the joint is the fulcrum or ſupport. The oars of a boat, the rudder of a ſhip, cutting knives fixed at one end, are levers of the ſecond kind. Tongs, ſheers, and the bones of animals, are levers of the third kind, a ladder raiſed upright, is a lever of the third kind. A hammer drawing out a nail is a lever of the fourth kind. Fig. 32.

The *Steel Yard* is nothing but a lever of the firſt kind, whoſe mechaniſm or conſtruction is this. Let AB be the beam, C the point of ſuſpenſion; P the power, movable along the beam CB. The beam being ſuſpended at D, move the power P, along towards C, till you find the point O, where P reduces the beam to an equilibrium. Then at A hang on the weight W of 1 pound; and move P to be in equilibrio with it at 1; then hang on W of 2 pound, and ſhift P till it be in equilibrio, at 2. Proceed thus with 3, 4, 5, &c. pounds at W, and find the diviſions 3, 4, 5, &c. Or if you will; after having found the points O, 1; make the diviſions, 12, 23, 34, &c. each equal to O1. But for more exactneſs and expedition, having found the point O, where P makes the beam in equilibrio: hang on any known number of pounds, as W; and move P to the point B, where it makes an equilibrium. Then divide OB into as many equal parts as W conſiſts of pounds: mark theſe diviſions 1, 2, 3, 4, &c. Then any weight W being ſuſpended at A. If P be placed to make an equilibrium therewith; then the number where P hangs will ſhew the pounds or weight of W.

To prove this, we muſt obſerve, that AC is the heavier end of the beam; therefore let Q be the *Momentum* at that end to make an equilibrium with P ſuſpended at O; that is, let $Q = CO \times P$. But (Cor. 2. Prop. XXXVI.) $Q + CA \times W = CF \times P = CO \times P + OF \times P$. Take away $Q = CO \times P$, and then $CA \times W = OF \times P$. Whence $AC : P :: OF : W$. But AC and P are always the ſame; therefore W is as OF; that is,

if

Fig. if OF be 1, 2, 3, &c. diviſions, then W is 1, 2, 3,
32. &c. pounds.

We may take notice that the diviſions O1, 12, 23, &c. are all equal; but CO may be greater or leſſer, or nothing.

If you would know how much the weight P is, take the diſtance CA, and ſet it from O along the diviſions O, 1, 2, 3, &c. and it will reach to the number of pounds. But this is of no conſequence, being only matter of curioſity.

PROP. XLVI.

33. *In the compound lever, or where ſeveral levers act upon one another, as* AB, BC, CD, *whoſe ſupports are* F, G, I; *the power* P : *is to the weight* W : : *as* BF × CG × DI : *to* AF × BG × CI.

For the power P acting at A : force at B : : BF : AF; and force or power at B : force at C : : CG : GB; and force or power at C : weight W : : DI : IC. Therefore *ex equo*, power P : weight W : : BF × CG × DI : AF × GB × IC.

And it is the ſame thing in the other kinds of levers, taking the reſpective diſtances, from the ſeveral props or ſupports.

PROP. XLVII.

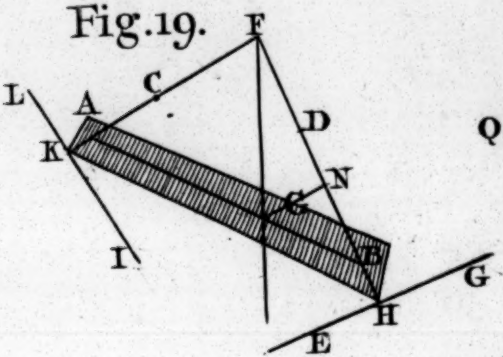
34. *In the wheel and axle; the weight and power will be in equilibrio, when the power* P *is to the weight* W; *as the radius of the axle* CA, *where the weight hangs; to the radius of the wheel* CB, *where the power acts.*

This is a wheel fixed to a cylindrical roller, turning round upon a ſmall axis; and having a rope going round it.

Thro'

Mechan

Fig.19.



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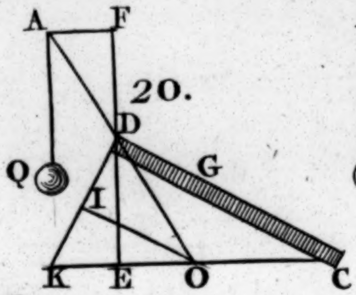

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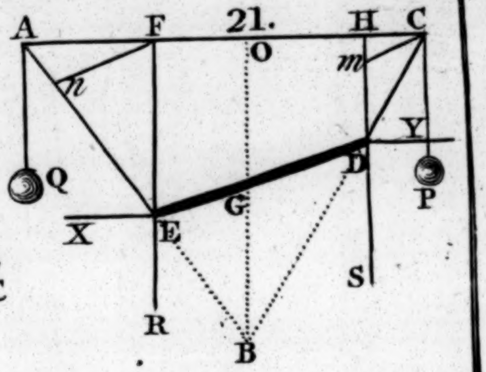

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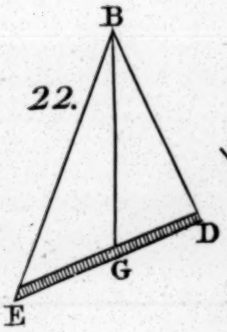

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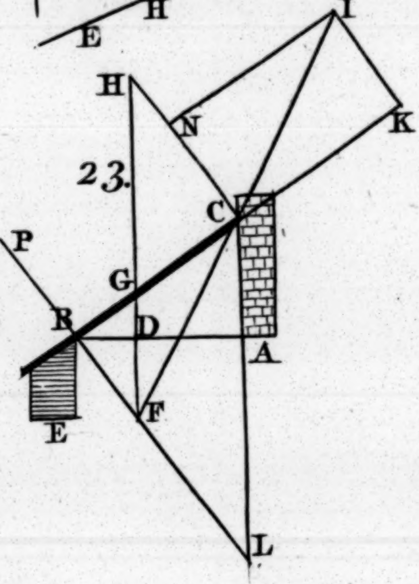

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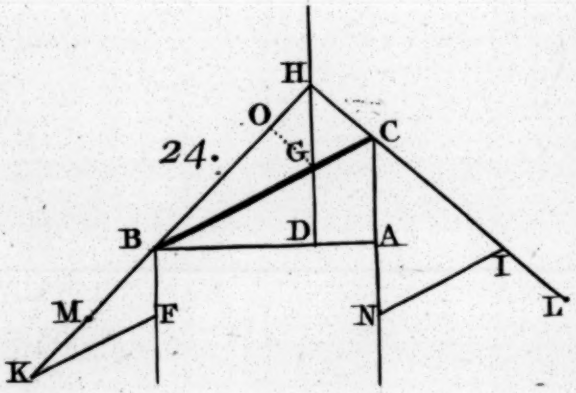

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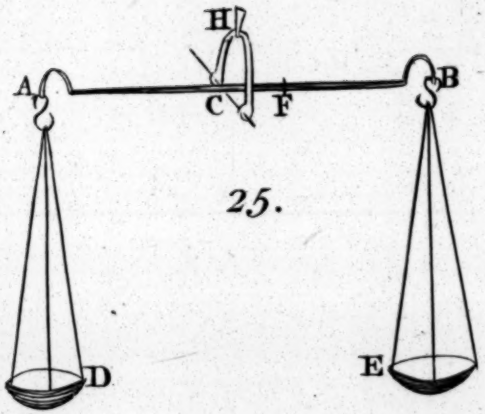

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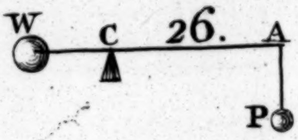

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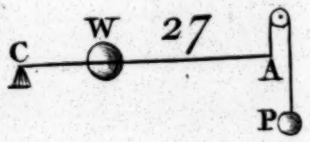

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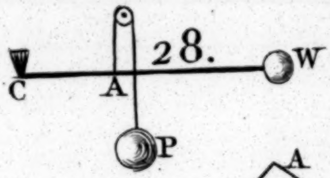

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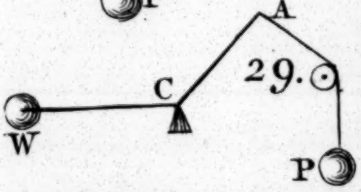

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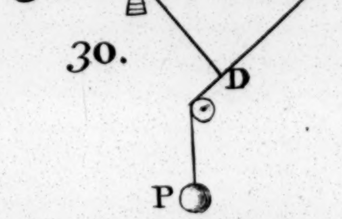

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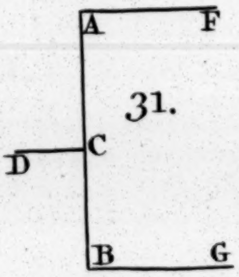

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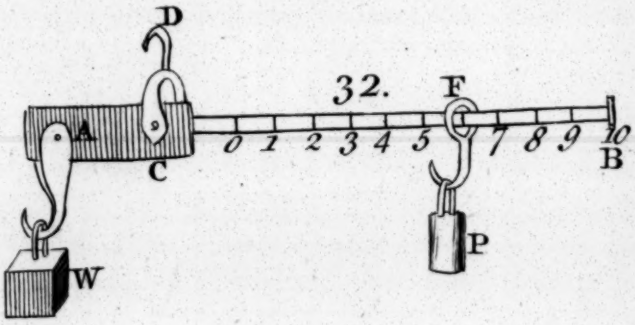


Thro' the center of the wheel C, draw the horizontal line BCA. Then BP and AW are perpendicular to BA; and BCA will be a lever whoſe ſupport is C; and the power acts always at the diſtance BC, and the weight at the diſtance CA; which remain always the ſame. Therefore the weight and power act always upon the lever BCA. But by the property of the lever (Prop. XLV.) BC : CA : : W : P, to have an equilibrium. Fig. 34.

Otherwiſe,

If the wheel be ſet a moving the velocity of the point A or of W, is to that of B or P, as CA to CB; that is (by ſuppoſition), as P : W. Therefore W × velocity of W = P × velocity of P; therefore the motions of P and W, being equal, they cannot, when at reſt, move one another.

Cor. 1. *If the power acting at the radius* CB, *act not at right angles to it; draw* CD *perpendicular to* BP *the direction of the power; then the power* P : *is to the weight* W : : *as the radius of the axle* CA : *to the perpendicular* CD. 35.

For in the lever DCA, whoſe ſupport is C, the power P : weight W : : CA : CD.

Cor. 2. *In a roller turned round, on the axis or ſpindle* FC, *by the handle* CBG; *the power applied perpendicularly to* BC *at* B, *is to the weight* W : : *as the radius of the roller* DA, *to the length of the handle* CB. 36.

For in turning round, the point B deſcribes the circumference of a circle; the ſame as if it was a wheel whoſe radius is CB.

SCHOLIUM.

All this is upon ſuppoſition that the rope ſuſtaining the weight is of no ſenſible thickneſs. But if it is a thick rope, or if there be ſeveral folds of it

Fig. 36. it about the roller or barrel; you muſt meaſure to the middle of the out ſide rope to get the radius of the roller. For the diſtance of the weight from the center is increaſed ſo much, by the rope's going round.

From hence the effects of ſeveral ſorts of machines, or inſtruments, may be accounted for. A roller and handle for a well or a mine, is the ſame thing as a wheel and axle, a windleſs and a capſtain in a ſhip is the ſame; and ſo is a crane to draw up goods with. A gimblet and an auger to bore with, may be referred to the wheel and axle.

The wheel and axle has a particular advantage over the lever; for a weight can but be raiſed a very little way by the lever. But by continual turning round of the wheel and roller, the weight may be raiſed to any hight required.

PROP. XLVIII.

37. *In a combination of wheels with teeth; if the power* P *be to the weight* W : : *as the product of the diameters of all the axles or pinions, to the product of the diameters of all the wheels; the power and weight will be in equilibrio.*

AC, CD are the radii of one wheel and its axle; DG, GH, the radii of another; and HI, IK are thoſe of another. Theſe act upon one another at D and H, then as the power or force P is propagated thro' all the wheels and axles to W; we muſt proceed to find the ſeveral forces acting upon them, by Prop. XXXVII. Thus,

$$\text{CD} : \text{CA} : : \text{P} : \frac{\text{CA}}{\text{CD}}\text{P} = \text{force acting at D.}$$

$$\text{and GH} : \text{GD} : : \frac{\text{CA}}{\text{CD}}\text{P (force at D)} : \frac{\text{CA}\times\text{GD}}{\text{CD}\times\text{GH}}\text{P}$$

$$= \text{force acting at H.}$$

and

Fig. 37.

and IK : IH : : $\frac{CA \times GD}{CD \times GH}$ P (force at H) : $\frac{CA \times GD \times IH}{CD \times GH \times IK}$ P = force at K = W. And $CA \times GD \times IH \times P = CD \times GH \times IK \times W$; whence P : W : : $CD \times GH \times IK$: : $CA \times GD \times IH$.

Cor. 1. *If the weight and power be in equilibrio, and made to move; the velocity of the weight, is to the velocity of the power; as the product of the diameters of all the axles or pinions, to the product of the diameters of all the wheels. Or inſtead of the diameters, take the number of teeth in theſe axles and wheels that drive one another. And the ſame is true of wheels carried about by ropes.*

For the power is to the weight; as the velocity of the weight to the velocity of the power. And the number of teeth in the wheels and pinions, that drive one another, are as the diameters. And the ropes ſupply the place of teeth.

Cor. 2. *In a combination of wheels with teeth. The number of revolutions of the firſt wheel, is to the number of revolutions of the laſt wheel, in any time; as the product of the diameters of the pinions or axles, to the product of the diameters of the wheels: or as the product of the number of teeth in the pinions, to the product of the number of teeth in the wheels which drive them. And the ſame is true of wheels going by cords.*

For as often as the number of teeth in any pinion, is contained in the number of teeth of the wheel that drives it; ſo many revolutions does that pinion make for one revolution of the wheel.

SCHOLIUM.

A pinion is nothing but a ſmall wheel, fixed at the other end of the axis, oppoſite to the wheel;

Fig. 37. and conſiſts but of a few leaves or teeth; and therefore is commonly leſs than the wheel. But in the ſenſe of this propoſition, a pinion may, if we pleaſe, be bigger than the wheel. As if we put the power and weight into the contrary places, the wheels will become the pinions, and the pinions the wheels, according to the meaning of this propoſition.

PROP. XLIX.

If a power ſuſtains a weight by means of a fixed pulley; the power and weight are equal: but if the pulley be movable along with the weight, then the weight is double the power.

A pulley is a ſmall wheel of wood or metal, turning round upon an axis, fixed in a block; on the edge of the pulley is a groove for the rope to go over.

38. 39. Thro' the centers of the pullies, draw the horizontal lines AB, CD; then will AB repreſent a lever of the firſt kind, and its ſupport is the center of the pulley, which is a fixed point, the block being fixed at F. And the points A, and B, where the power and weight act, being equally diſtant from the ſupport, therefore (Prop. XLV.) the power P = weight W.

Alſo CD repreſents a lever of the ſecond kind, whoſe ſupport is at C, a fixed point; the rope CG being fixed at G. And the weight W acting at the middle of CD, and the power acting at D, twice the diſtance from C; therefore (Prop. XLV.) the power P is to the weight W : : as $\frac{1}{2}$ CD to CD; or as 1 to 2.

Cor. *Hence all fixed pulleys are levers of the firſt kind, and ſerve only to change the direction of the motion; but make no addition at all to the power. And*

And therefore if a rope goes over ſeveral fixed pullies; the power is not increaſed, but rather decreaſed, by the friction. Fig. 38. 39.

SCHOLIUM.

The uſe of a fixed pulley is of great ſervice in raiſing a weight to any height, which otherwiſe muſt be carried by ſtrength of men, which is often impracticable. Therefore if a rope is fixed to the weight at W (fig. 38.) and paſſed over the pulley BA; a man taking hold at P will draw up the weight, without moving from the place. And if the weight be large, ſeveral perſons may pull together at F, to raiſe the weight up; where in many caſes they cannot come to it, to raiſe it by ſtrength.

PROP. L.

In a combination of pullies, all drawn by one rope going over all the pullies; if the power P *is to the weight* W; *as* 1 *to the number of the parts of the rope proceeding from the movable block and pullies. Then the power and weight will be in equilibrio.* 40.

Let the rope go from the power about the pullies in this order, *ntovrs*, where the laſt part *s* is fixed to the lower block B. Now (Ax. 13.) all the parts of the rope *ntovrs* are equally ſtretched, and therefore each of them bears an equal weight; but the part *n* bears the power P, which goes to the fixed block A. All the other parts, ſuſtain the weight and movable block B, each with a force equal to P. Therefore P is to the ſum of all the forces, ſuſtained by *o*, *r*, *s*, *t*, *v*, or the weight W, as 1 to the number of theſe ropes immediately communicating with the movable block B. And all the ropes having an equal tenſion, none of

Fig. 40. them can move the reſt, but they muſt remain in equilibrio.

And if you take away the power at P, and apply a force at the rope *t* equal to P, to pull upwards in direction *t*A; this will make no alteration, for the rope *t* draws from the movable block with the ſame force as before, and therefore the weight is ſuſtained as before; for the upper pulley (by Prop. XLIX. Cor.) which the rope *nt* goes over, ſerves only to change the direction. And therefore as there are the ſame number of ropes ſtill drawing from the movable block as before; the propoſition holds good alſo in this reſpect. And it would be the ſame thing if the rope *s* was fixed to the weight W inſtead of the block B; but had it been fixed to the block A, there muſt have been a pulley more below, and a rope more, which would have increaſed the power, according to the propoſition.

Cor. 1. *Hence it appears to be a diſadvantage to the power to pull againſt the fixed block.*

For the rope *n* has no more purchaſe, or no more effect than the rope *t* has which draws againſt the movable block; and therefore when one draws by the rope *n*, there muſt be a pulley more, which will create more friction.

Cor. 2. *Hence one may explain the effects of all ſorts of machines compoſed of pullies; or find out ſuch a conſtruction or combination of them as to anſwer any purpoſe deſired. And to find its force, begin at the power, and call it* 1; *then all parts of the running rope that go and return about ſeveral pullies, muſt be each numbered alike. And any rope that acts againſt ſeveral others muſt be numbered with the ſum of theſe. And ſo on to the weight.*

For

Fig. 41. *For example*; ſuppoſe a man wanted to draw himſelf up to the top of a houſe or a church. Get a pulley A fixed at the top, and place another B at the bottom. Let a rope be fixed to the upper block A, and brought down about the pulley B, and then put round the upper pulley, and ſo brought to the ground at H. Then if a croſs ſtick CD be faſtened to the block B by a rope; a man may get aſtride of the ſtick, and then draw himſelf up by the rope H. And the power to draw himſelf up, will be little more than $\frac{1}{3}$ of his weight. For the power at H, and the two parts of the rope going about the pulley B, ſuſtain all his weight; and each of them ſuſtains one third of it.

If inſtead of the ſtick CD, he takes a chair to ſit in; then when he has drawn himſelf up to any hight he pleaſes, he may fix the rope H to the chair, and then do any ſort of buſineſs, as ſet up a dial, point the walls, and ſuch like, as is commonly done.

42. Again, ſeveral tackles are uſed aboard a ſhip, for hoiſting goods and the like. Let A, B be two blocks with pullies, the upper one being fixed, and let a weight W be ſuſpended at the ſingle pulley and rope, one end of the rope being fixed at F, and the other faſtened to the movable block B. This pulley and rope BCF is called a *Runner*. Let the power be at P, call it 1; then all the ropes going from B to A, muſt be each of them 1, and the rope going from the block B, acting againſt theſe four muſt be marked 4, and the other part of it CF muſt alſo be 4. Laſtly, the weight acting againſt theſe two, muſt be 8. And then the power P is to the weight W, as 1 to 8.

43. ABCD is another tackle with a runner BAD, A being a fixed pulley; the two blocks B, C, are both movable. The rope DAB is fixed to the weight

Fig. 43. weight at D, and to the block B. The rope PB goes and returns about the pullies BC, and at laſt is faſtened to the block C. Let P be the power, mark it 1, then the other parts of the rope between the blocks, muſt alſo be 1 apiece. Then CI acting againſt 3, muſt be 3. And AB is 4, as it acts againſt 4; likewiſe AD muſt be 4. Therefore the whole force that ſuſtains the weight W is 3 and 4, or 7. And the power to the weight as 1 to 7.

44. The following is a ſort of *Spaniſh burton*, A and F are two fixed pullies; C and B two movable ones. The rope going from the power P, goes round C, B, and A, and is faſtened to the block B. Another rope is faſtened to the block B, and goes over the pulley F, and is fixed to the block C. Then marking the power P, 1. Then each part of the rope, continued over C, B, and A to B again muſt be each 1. Then FC muſt be 2, as it acts againſt two parts; and likewiſe the other part of it FB muſt be 2. Then the whole that lifts the weight W, is 1 + 1 + 1 + 2 = 5. And therefore the power is to the weight as 1 to 5.

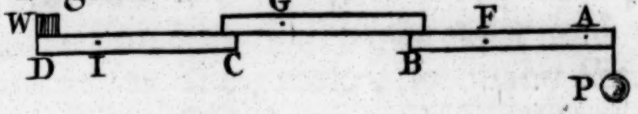
The friction between the pullies and blocks is ſometimes conſiderable. To remedy which, they muſt be as large as they can conveniently be made, and kept oiled or greaſed.

PROP. LI.

45. *In the ſcrew, if the power applied at* E, *be to the weight, preſſure, &c. at* B; *as the diſtance of two threads of the ſcrew, taken parallel to the axis of it, is to the circumference deſcribed by the power at* E; *then the weight and power will be in equilibrio.*

A ſcrew is an inſtrument conſiſting of two parts AB, CD, fitting into one another. AB is the male ſcrew, called the top or ſpindle; this is a long cylindrical body, having its ſurface cut into ridges and

Fig. 33.



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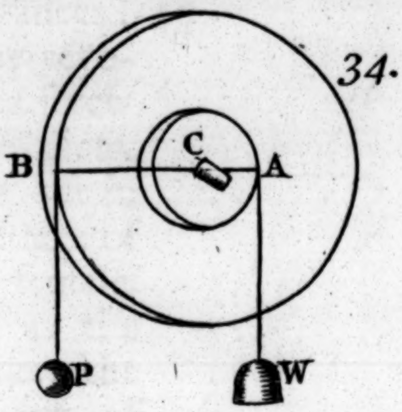

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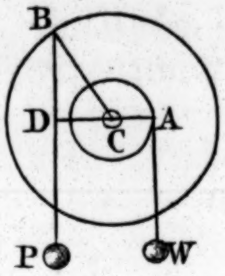

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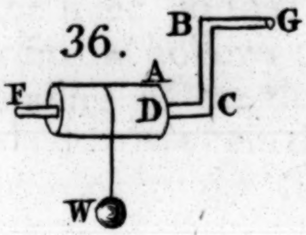

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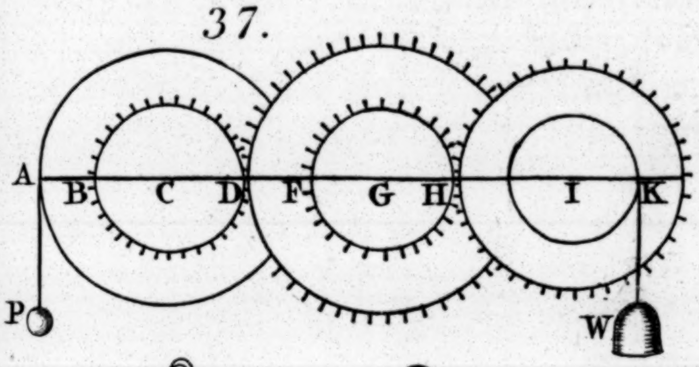

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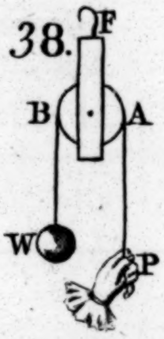

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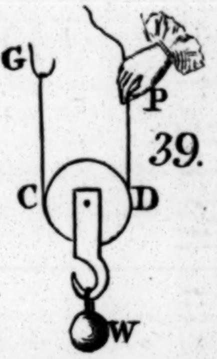

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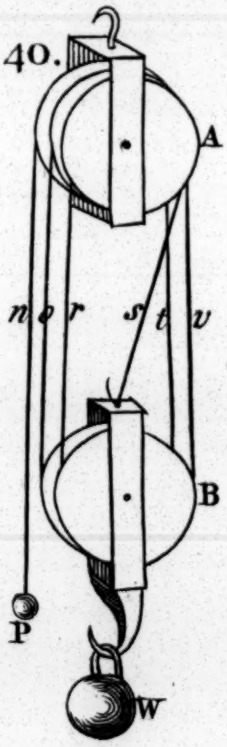

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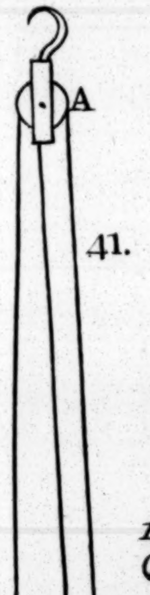

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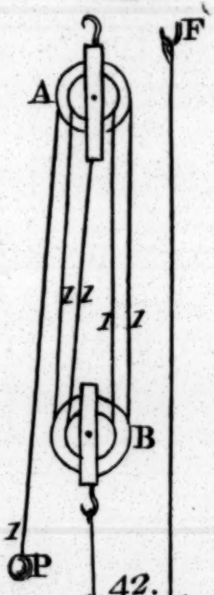
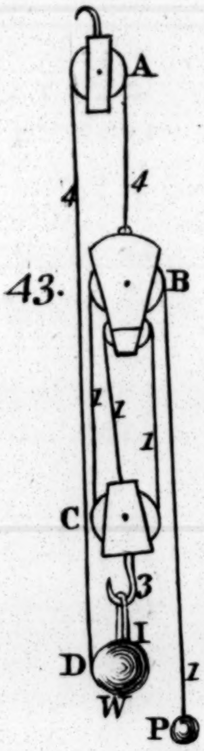
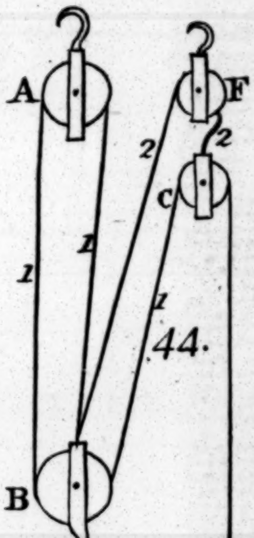


43.



44.



Mechanics

Pl. III. pa. 74.

and hollows, that run round it in a ſpiral manner from one end to the other, at equal diſtances; theſe riſings are called threads, and ſo many revolutions as they make, ſo many threads the ſcrew contains. CD is the female ſcrew, or the plate, thro' which the other goes; its concavity is cut in the ſame manner as the male, ſo that the ridges of the male may exactly fit the hollows of the female. By reaſon of the winding of the threads, as the handle EF is turned one way or the other, the male AB goes further in or comes further out, of the female. Fig. 45.

Let the point E of the handle, make one revolution, then the male AB will have advanced the diſtance between one thread and another, of the ſcrew. Therefore if G repreſent any weight, which the end B acts againſt, it will be moved thro' the breadth of a thread, whilſt the power moves thro' a circumference whoſe radius is EA. Therefore the velocity of G is to the velocity of E, as the breadth of a thread, to the circumference deſcribed by E; that is (by ſuppoſition) as the power at E, to the weight at G. Therefore E × velocity of E = G × velocity of G; and therefore their motions being equal, they will be in equilibrio.

Cor. *By reaſon of the friction, if any weight is to be removed by a ſcrew; the power muſt be to the weight; at leaſt as the breadth of two threads of the ſcrew, to the circumference deſcribed by the power; to keep the weight in equilibrio; and muſt be much more to move it.*

For in the ſcrew there is ſo much friction, that it will ſuſtain the weight when the power is taken away. And therefore the friction is as great or greater than the power. And therefore the whole power applied muſt at leaſt be doubled to produce any motion.

Fig. 45.

SCHOLIUM.

Screws with ſharp threads have far more friction than thoſe with ſquare threads, and therefore move a body with more difficulty.

The ſcrew, in moving a body, acts like an inclined plane. For it is juſt the ſame as if an inclined plane was forced under a body to raiſe it; the body being prevented from flying back, and the baſe of the plane being driven parallel to the horizon.

The uſe of this power is very great. It is of great ſervice for fixing ſeveral things together by help of ſcrew nails; it is likewiſe very uſeful for ſqueezing or preſſing things cloſe together, or breaking them; alſo for raiſing or moving large bodies. The ſcrew is uſed in preſſes for wine, oil, or for ſqueezing the juice out of any fruit. The very friction of this machine has its particular uſe, for when a weight is raiſed to any hight; if the power be taken away, the ſcrew will retain its poſition, and hinder the weight from deſcending again by its friction, without any other power to ſuſtain it.

In the common ſcrew, ſuch as is here ſuppoſed; the threads are all one continued ſpiral from one end to the other; but where there are two or more ſpirals, independent of one another, as in the worm of a jack; you muſt meaſure between thread and thread of the ſame ſpiral, in computing the power.

PROP.

Fig.

PROP. LII.

In the endleſs ſcrew, where the teeth of the worm or ſpindle AB, *drives the wheel* CD, *by acting againſt the teeth of it. If the power applied at* P, *is to the weight* W, *acting upon the edge of the wheel at* C : : *as the diſtance of two threads or teeth, between fore ſide and fore ſide, taken along* AB; *is to the circumference deſcribed by the power* P. *Then the weight and power will be in equilibrio.* 46.

The *endleſs* or *perpetual ſcrew* is one that turns perpetually round the axis AB; and whoſe teeth fit exactly into the teeth of the wheel CD, which are cut obliquely to anſwer them. So that as AB turns round, its teeth take hold of the teeth of the wheel CD, and turns it about the axis I, and raiſes the weight W.

For by one revolution of the power at P, the wheel will be drawn forward one tooth; and the weight W will be raiſed the ſame diſtance. Therefore the velocity of the power, will be to that of the weight; as that circumference, to one tooth : : that is (by ſuppoſition) as the weight W, to the power P. Therefore the power P × velocity of P = W × velocity of W; therefore their motions being equal, they will be in equilibrio.

Cor. *If a weight* N *be ſuſpended at* E *on the axle* EF; *then if the power* P, *is to the weight* N : : *as the breadth of a tooth* × EF, *to the circumference deſcribed by* P × CD. *They will be in equilibrio.*

Or if the power P; *is to the weight* N : : *as radius of the axle* EI, *to the radius of the handle* FP × *by the number of teeth in* CD; *they will be in equilibrio.*

For

Fig. For power P : weight W : : 1 tooth : circumference.
46. and weight W : weight N : : EF : CD.
therefore P : N : : 1 tooth × EF : CD × circumference.

Or thus, whilſt EF turns round once, P turns round as oft as CD has teeth; whence EI : BP × number of teeth : : velocity of N : velocity of P : : P : N.

SCHOLIUM.

47. As the teeth of the wheel CD, muſt be cut obliquely to anſwer the teeth or ſcrew on AB; ſuppoſing AB to lie in the plane of the wheel CD; and therefore the wheel will be acted on obliquely by the ſcrew AB. To remedy that, the ſcrew AB may be placed oblique to the wheel, in ſuch a poſition, that when the teeth of the wheel are cut ſtreight or perp. to its plane, the teeth of the ſcrew AB, may coincide with them, and fit them. By that means the force will be directed along the plane of the wheel CD. Fig. 47 explains my meaning.

This machine is of excellent uſe, not only in itſelf, for raiſing great weights, and other purpoſes; but in the conſtruction of ſeveral ſorts of compound engines.

PROP. LIII.

48. *In the wedge* ACD, *if a power acting perpendicular to the back* CD, *is to the force acting againſt either ſide* AC, *in a direction perpendicular to it; as the back* CD, *to either of the ſides* AC; *the wedge will be in equilibrio.*

A wedge is a body of iron or ſome hard ſubſtance in form of a priſm, contained between two iſoceles triangles, as CAD. AB is the hight, and CD the back of it; AC, AD the ſides.

Let AB be perp. to the back CD, and BE, BF, perp. to the ſides AC, AD. Draw EG, FG parallel

Fig. 48.

rallel to BF, BE; then all the ſides of the parallelogram BEGF are equal. The triangles EGB, ADC are ſimilar; for draw EOF which will be perp. to AB; then the right angled triangles AEB, AEO, are ſimilar, and the angle ABE = AEO = ACB; that is, GBE = ACD, and likewiſe BGE = ADC, whence CAD = BEG.

Now let BG be the force acting at B, in direction BA, perp. to CD; then (Prop. IX.) the forces againſt the ſides AC, AD, will be in the directions EB, FB; and therefore EB, EG will repreſent theſe forces (by Prop. VIII.), when they keep one another in equilibrio. Therefore force BG applied to the back of the wedge, is to the force BE, perp. to the ſide AC; as BG to BE; that is, (by ſimilar triangles) as CD to CA.

Cor. 1. *The power acting againſt the back at* B, *is to that part of the force againſt* AC, *which acts parallel to the back* CD; *as the back* CD, *is to the bight* AB.

For divide the whole force BE into the two BO, OE; the part EO acts parallel to CD; therefore the force acting at B, is to the force in direction OE or BC; as BG to OE; that is, (by ſimilar triangles) as CD to AB.

Cor. 2. *By reaſon of the great friction of the wedge, the power at* B, *muſt be to the reſiſtance againſt one ſide* AC; *at leaſt as twice the baſe* CD, *to the ſide* AC, *taking the reſiſtance perp. to* AC. *Or as twice the baſe* CD, *to the bight* AB, *for the reſiſtance parallel to the baſe* CD; *to overcome the reſiſtance. But the power muſt be doubled for the reſiſtance againſt both ſides.*

For ſince the wedge retains any poſition it is driven into; therefore the friction muſt be at leaſt equal to the power that drives it.

Fig. 48. Cor. 3. *If you reckon the reſiſtance at both ſides of the wedge; then, if there is an equilibrium, the power at* B, *is to the whole reſiſtance; as the back* CD, *to the ſum of the ſides*, CA, AD, *reckoning the reſiſtance perp. to the ſides. Or as the back* CD, *to twice the hight* AB, *for the reſiſtance parallel to the back* CD.

This follows directly from the Prop. and Cor. 1.

SCHOLIUM.

The principal uſe of the wedge is for the cleaving of wood or ſeparating the parts of hard bodies, by the blow of a mallet. The force impreſſed by a mallet is vaſtly great in compariſon of a dead weight. For if a wedge, which is to cleave a piece of wood, be preſſed down with never ſo great a weight, or even if the other mechanical powers be applied to force it in; yet the effect of them will ſcarce be ſenſible; and yet the ſtroke of a ſledge or mallet will force it in. This effect is owing very much to the quantity of motion the mallet is put into, which it communicates in an inſtant to the wedge, by the force of percuſſion. A great deal of the reſiſtance is owing to friction, which hinders the motion of the wedge; but the ſtroke of a mallet overcomes it; upon which account the force of percuſſion is of excellent uſe; for a ſmart ſtroke puts the body into a tremulous vibrating motion, by which the parts are diſunited and ſeparated; and by this means the friction or ſticking is overcome, and the motion of the wedge made eaſy.

This mechanic power is the ſimpleſt of any; and to this, may be reduced all edge tools, as knives, axes, chiſſels, ſciſſars, ſwords, files, ſaws, ſpades, ſhovels, &c. which are ſo many wedges faſtened to a handle. And alſo all tools or inſtruments with a ſharp point, as nails, bodkins, needles, pins; and all inſtruments to cleave, cut, ſlit, chop, pierce, bore, and the like. And in general all inſtruments that have an edge or point. Fig. 48.

This Prop. is the ſame as Prop. XXX. in my large book of Mechanics, but demonſtrated after a different way; and both come to the ſame thing, which evinces the truth thereof.

In this Prop. I have ſhewn under what circumſtance, the wedge is in equilibrio; and that is, when the power is to the force againſt either ſide; as the back, is to that ſide. Therefore it muſt be very ſtrange, that any body ſhould underſtand it, as if I had ſaid, that the power is to the whole reſiſtance; as the back, is to one ſide only. They that do this muſt be blind or very careleſs.

Fig.

SECT. V.

The comparative Strength of Beams of Timber, and the Streſs they ſuſtain. The Powers of Engines, their Motions, and Friction.

PROP. LIV.

49. *If a beam of wood* AB, *whoſe ſection is a parallelogram, be ſupported at the ends* A *and* B, *by two props* C, D. *And a weight* E *be laid on the middle of it, to break it; the ſtrength of it will be as the ſquare of the depth* EF, *when the breadth is given.*

For divide the depth EF into an infinite number of equal parts at *n*, *o*, *p*, *q*, *r*, &c. Now the ſtrength of the beam conſiſts of the ſtrength of all the fibres F*n*, *no*, *op*, &c. And to break theſe fibres, is to break the beam. Alſo when the beam is ſtretched by the weight, the fibres *Fn*, *no*, *op*, &c. are ſtretched by the power of the bended levers AEF, AE*n*, AE*o*, &c. whoſe ſupport is at E, and power at A. For the preſſure at A being half the weight E, we muſt ſuppoſe that preſſure applied to A, to overcome the reſiſtances at F, *n*, *o*, &c. Put the force or preſſure at A = P, then P acts againſt all the fibres at F, *n*, *o*, &c. by help of the bended levers AEF, AE*n*, AE*o*, &c. But it is a known property of ſprings, fibres, and ſuch like expanding bodies; that the further they are ſtretched, the greater force they exert, in proportion to the length. Therefore when the beam breaks; that is, when the tenſion of the fibre F*n* is

Fig. 49.

is at its utmoſt extent; then thoſe in the middle between F and E will have but half the tenſion, and thoſe at all other diſtances, will have a tenſion proportional to that diſtance. This being ſettled, let the utmoſt tenſion of nF be $= 1$; then the tenſions at n, o, p, &c. will be $\frac{En}{EF}$, $\frac{Eo}{EF}$, $\frac{Ep}{EF}$ &c. and the ſeveral forces, theſe exert againſt the point A, by means of the bended levers FEA, nEA, oEA, &c. will be $\frac{FE}{EA}$, $\frac{En^2}{EF \times EA}$, $\frac{Eo^2}{EF \times EA}$, $\frac{Ep^2}{EF \times EA}$ &c. and the ſum of all is $= \frac{1}{EF \times EA} \times$ into $EF^2 + En^2 + Eo^2 + Ep^2$ &c. to o. But (Arith. Inf. Prop. III.) the ſum of the progreſſion $EF^2 + En^2 + Eo^2$ &c. to o, is $\frac{1}{3} EF^3$. Therefore the ſum of all the forces exerted at A; that is, $P = \frac{1}{EF \times EA} \times \frac{1}{3} EF^3 = \frac{EF^2}{3EA}$. But $P = \frac{1}{2}$ weight E, therefore weight $E = \frac{2}{3} \times \frac{EF^2}{EA}$, when the beam breaks.

In like manner for any other depth Ep, the weight e that would break it is $= \frac{2Ep^2}{3EA}$. Whence the weight E to the weight e, is as EF^2 to Ep^2; that is, as the ſquares of the depths; for $\frac{2}{3EA}$ is a given quantity. Therefore the ſtrength of the beams, are as the ſquares of the depths.

Cor. 1. *Hence the ſtrengths of ſeveral pieces of the ſame timber, are to one another as the breadths and ſquares of the depths.*

For by this Prop. they are as the ſquares of the depths when the breadth is given. And if the breadth be increaſed in any proportion, it is evi-

dent

Fig. 49. dent the ſtrength is increaſed in the ſame proportion. So that a beam of the ſame depth being twice as broad is twice as ſtrong, and thrice as broad is thrice as ſtrong, &c.

50. Cor. 2. *If ſeveral beams of timber as* AF *of the ſame length, ſtick out of a wall; their ſtrength to bear any weight* W *ſuſpended at the end, is as the breadth and ſquare of the depth.*

This follows from Cor. 1. only turning the beam upſide down, to make the weight W ſuſpended at A act downwards inſtead of preſſing upwards.

49. Cor. 3. *If ſeveral pieces of timber be laid under one another, they will be no ſtronger, than if they were laid ſide by ſide.*

For not being connected together in one ſolid piece, they can only exert each its own ſtrength, which will be the ſame in any poſition.

Cor. 4. *Hence the ſame piece of timber is ſtronger when laid edgeways or with the flat ſide up and down, than when laid flat ways, or with the flat ſide horizontal; and that in proportion of the greater breadth to the leſſer.*

For let B be the greater breadth, or the breadth of the flat ſide; *b* the leſſer breadth, being the narrow ſide. Then the ſtrength edge ways is BB*b*, and flat ways B*bb*; and they are to one another as B to *b*.

PROP. LV.

49. *If a beam of timber* AB *be ſupported at both ends; and a given weight* E *laid on the middle of it; the ſtreſs it ſuffers by the weight, will be as its length* AB.

For half the weight E is ſupported at A, by the prop C; and the preſſure at C is equal to it. And this preſſure is always the ſame whatever length

AB is of. But it was ſhewn in the laſt Prop. that the preſſure at A, breaks the fibres F*n*, *no*, *op*, &c. by means of the bended levers AEF, AE*n*, AE*o*, &c. But (by Prop. XLV.) when the lengths EF, E*n*, E*o*, &c. are given, and the power at A alſo given; the effect at F, *n*, *o*, &c. is ſo much greater, as the arm AE is longer; that is, the ſtreſs at the ſection EF, is proportional to the diſtance AE, or to the length of the beam AB. Fig. 49.

Cor. 1. *If* AF *be a beam ſticking out of a wall, and a weight* W *hung at the end of it. The ſtreſs it ſuffers by the weight, at any point* G, *will be as the diſtance* AG. 50.

For this has the ſame effect, as in the caſe of this Prop. only turning the beam upſide down. Or thus, ſuppoſe AHG to be a bended lever, whoſe fulcrum is H; then ſince GH is given, and the weight W; therefore by the power of the lever, the longer AH is, the more force is applied at G, or any other points, in GH, to ſeparate the parts of the wood; and therefore the ſtreſs is as AG.

Cor. 2. *Therefore, inſtead of a weight, if any force be applied at the end* A, *of the lever* AF; *the ſtreſs at any part* G, *will be as the force, and diſtance* AG.

For augmenting the force, the ſtreſs is increaſed in the ſame ratio.

Cor. 3. *Hence alſo if any weight lie upon the middle of a horizontal beam; the ſtreſs there will be as the weight and length of the beam.* 49.

For if the weight be increaſed, the ſtreſs will be increaſed proportionally, all other circumſtances remaining the ſame.

Cor. 4. *The ſtreſs of beams by their own weight, will be as the ſquares of the lengths.*

For here the weight is as the length,

Fig.

PROP. LVI.

51. *If* AB *be a beam of timber whoſe length is given; and ſupported at the ends* A *and* B; *and if a given weight* W *be placed at any point of it* G. *The ſtreſs of the beam at* G, *will be as the rectangle* AGB.

Let the given weight be W, then (Cor. 3. Prop. XLV.) the weight W is equal to the preſſure at both A and B. And (Cor. 2. Prop. XXXI.) preſſure at A : preſſure at B : : BG : AG, and preſſ. A : preſ. A + preſ. B : : BG : BG + AG; that is, preſ. A : weight W : : BG : AB, therefore preſſure at A $= \frac{BG}{AB}W$, and this is the force re-acting at A. But (Prop. LV. Cor. 2.) the ſtreſs at G by this force acting at the diſtance AG, is as the force multiplied by AG; that is, as $\frac{AG \times BG}{AB} \times W$. But W and AB are given, and therefore the ſtreſs at G is as $AG \times GB$.

Cor. 1. *The greateſt ſtreſs of a beam is when the weight lies in the middle.*

For the greateſt rectangle of the parts, is in that point.

Cor. 2. *The ſtreſs at any point* P *by a weight at* G; *is equal to the ſtreſs at* G, *by the ſame weight at* P.

For when the weight is at W, the ſtreſs at G is $AG \times GB$, and the ſtreſs at P $= \frac{BP}{BG} \times$ the ſtreſs at G $= \frac{BP}{BG} \times AG \times GB = BP \times AG$. Again, when the weight is at P, the ſtreſs at P is $AP \times PB$; and the ſtreſs at G $= \frac{AG}{AP} \times$ laſt ſtreſs $= \frac{AG}{AP} \times AP \times PB = AG \times PB$, the ſame as before.

PROP.

PROP. LVII.

If the diſtance of the walls AD *and* BC *be given,* 52.
and AB, AC *be two beams of timber of equal thickneſs; the one horizontal, the other inclined. And if two equal weights* P, Q, *be ſuſpended in the middle of them; the ſtreſs is equal in both, and the one will as ſoon break as the other, by theſe equal weights.*

For (Prop. XIV.) $AC : AB :: \text{weight } P : \frac{AB}{AC}P$ = preſſure againſt the plane, or the part of the weight the beam AC ſuſtains. And (Cor. 3. Prop. LV.) the ſtreſs upon AC is $\frac{AB}{AC}P \times AC$ or $AB \times P$; and the ſtreſs on AB is $Q \times AB$, which is equal to $AB \times P$, becauſe the weights P, Q are equal. Therefore, the ſtreſs being the ſame, and the beams being of equal thickneſs, one will bear as much as the other, and they will both break together.

Cor. 1. *If the beams be loaded with weights in any other places in the ſame perpendicular line as* F, G; *they will bear equal ſtreſs, and one will as ſoon break as the other.*

For they are cut into parts ſimilar to one another; and therefore ſtreſs at F : ſtreſs by P :: $AFC : \frac{1}{4}AC^2 :: AGB : \frac{AB^2}{4}$ :: ſtreſs by B : ſtreſs by Q or ſtreſs by P. Therefore ſtreſs at F = ſtreſs at B.

Cor. 2. *If the two beams be loaded in proportion to their lengths; the ſtreſs by theſe weights, or by their own weights, will be as their lengths; and therefore the longer, that ſtands aſlope, will ſooner break.*

Fig. 52. For the ſtreſs upon AC was AB × P, and the ſtreſs on AB was AB × Q; but ſince P and Q are to one another as AC and AB, therefore the ſtreſs on AC and AB will be as AB × AC and AB × AB; that is, as AC to AB. And in regard to their own weights, theſe are alſo proportional to their lengths.

PROP. LVIII.

53. *Let* AB, AC, *be two beams of timber of equal length and thickneſs, the one horizontal the other ſet ſloping. And if* CD *be perp. to* AB, *and they be loaded in the middle with two weights* P, Q, *which are to one another as* AC *to* AD. *Then the ſtreſs will be equal in both, and one will as ſoon break as the other.*

For (Prop. XIV.) $AC : AD :: P : \frac{AD}{AC} P =$ preſſure of P in the middle of AC. And by ſuppoſition, $AC : AD :: P : Q$; therefore $\frac{AD}{AC} P =$ Q, the weight in the middle of AB. Therefore the forces in the middle of the two beams are the ſame; and the lengths of the beams being the ſame, therefore (Prop. LV.) the ſtreſs is equal upon both of them; and being of equal thickneſs, if one breaks the other will break.

Cor. *If the weights* P, Q, *be equal upon the two equal beams* AB, AC. *The ſtreſs upon* AB *will be to the ſtreſs upon* AC, *as* AB *or* AC *to* AD. *The ſame holds in regard to their own weights.*

For the weight Q is increaſed in that proportion.

SCHOLIUM.

Many more propoſitions relating to the ſtrength of timber might be inſerted; as for example, if a weight was diſpoſed equally thro' the length of the beam AB (fig. 51.), ſupported at both ends; the ſtreſs in any point G, is as the rectangle AGB. And the ſtreſs at any point G is but half of the ſtreſs it would ſuffer, if the whole weight was ſuſpended at G. Alſo if AF (fig. 50.) be a beam fixed in a wall at one end, and a weight be diſperſed uniformly thro' all the length of it. The ſtreſs at any point G, with that weight (or with its own weight, if it be all of a thickneſs), will be as AG ſquare, the ſquare of the diſtance from the end. And the ſtreſs at any point G by a weight ſuſpended at A, will be double the ſtreſs at the ſame point G, when the ſame weight is diſperſed uniformly thro' the part AG. They that would ſee theſe and ſuch like things demonſtrated, may conſult my large book of Mechanics, to which I refer the reader. Fig. 53.

PROP. LIX.

If ſeveral pieces of timber be applied to any mechanical uſe where ſtrength is required; not only the parts of the ſame piece, but the ſeveral pieces in regard to one another, ought to be ſo adjuſted for bigneſs; that the ſtrength may be always proportional to the ſtreſs they are to endure.

This Prop. is the foundation of all good Mechaniſm, and ought to be regarded in all ſorts of tools and inſtruments we work with, as well as in the ſeveral parts of any engine. For who that is wiſe, will overload himſelf with his work tools, or make them bigger and heavier than the work requires? neither ought they to be ſo ſlender as not to be able to perform their office. In all engines, it muſt be conſidered what weight every beam is to carry, and proportion the ſtrength accordingly. All levers

Fig. vers muſt be made ſtrongeſt at the place where they are ſtrained the moſt; in levers of the firſt kind, they muſt be ſtrongeſt at the ſupport. In thoſe of the ſecond kind, at the weight. In thoſe of the third kind, at the power, and diminiſh proportionally from that point. The axles of wheels and pullies, the teeth of wheels, which bear greater weights, or act with greater force, muſt be made ſtronger. And thoſe lighter, that have light work to do. Ropes muſt be ſo much ſtronger or weaker, as they have more or leſs tenſion. And in general, all the parts of a machine muſt have ſuch a degree of ſtrength as to be able to perform its office, and no more. For an exceſs of ſtrength in any part does no good, but adds unneceſſary weight to the machine, which clogs and retards its motion, and makes it languid and dead. And on the other hand, a defect of ſtrength where it is wanted, will be a means to make the engine fail in that part, and go to ruin. So neceſſary it is to adjuſt the ſtrength to the ſtreſs, that a good mechanic will never neglect it; but will contrive all the parts in due proportion, by which means they will laſt all alike, and the whole machine will be diſpoſed to fail all at once. And this will ever diſtinguiſh a good mechanic from a bad one, who either makes ſome parts ſo defective, imperfect and feeble as to fail very ſoon; or makes others, ſo ſtrong or clumſey, as to out laſt all the reſt.

From this general rule follows

Cor. 1. *In ſeveral pieces of timber of the ſame ſort, or in different parts of the ſame piece; the breadth multiplied by the ſquare of the depth, muſt be as the length multiplied by the weight to be born.*

For then the ſtrength will be as the ſtreſs.

Cor. 2. *The breadth mutiplied by the ſquare of the depth, and divided by the product of the length and weight, muſt be the ſame in all.*

Cor. 3. *Hence may be computed the ſtrength of timber proper for ſeveral uſes in building. As,*

1. To find the dimenſions of joiſts and boards for flooring. Let b, d, l be the breadth, depth and length of a joiſt, n = number of them, x = their diſtance, g = depth of a board, w = weight; then $nbdd$ = ſtrength of all the joiſts, and wl = ſtreſs on them, alſo $nlgg$ = ſtrength of the boards, and wx their ſtreſs; therefore $\frac{nbdd}{wl} = \frac{nlgg}{wx}$; and $x = \frac{llgg}{bdd}$, for the diſtance of the joiſts, or the length of a board between them. Or $b = \frac{llgg}{ddx}$, or $dd = \frac{llgg}{bx}$, and ſo on, according to what is wanted.

2. To find the dimenſions of ſquare timber for the roof of a houſe. Let r, s, l be the length of the ribs, ſpars and lats, ſo far as they bear; x, y, z their breadth or depth, n the diſtances of the lats, w = weight upon a rib, c = coſine of elevation of the roof. Then by reaſon of the inclined plane, $\frac{lw}{r} \times c$ = weight upon a ſpar. And $\frac{lnw}{rs}$ = weight upon a lat: for the ribs and lats lie horizontally. Therefore (Cor. 2.) $\frac{x^3}{wr} = \frac{y^3}{\frac{slw}{r} \times c} = \frac{z^3}{l \times \frac{lnw}{rs}}$. Whence $x^3 = \frac{rry^3}{cls}$, and $x^3 = \frac{rrsz^3}{lln}$. Hence if any one x, y, or z be given, and all the reſt of the quantities; the other two may be found. Or in general, any two being unknown, they may be found, from having the reſt given

For example, let $r = 9$ feet, $s = 4$ feet, $l = 15$ inches, $n = 11$ inches, $c = .707$ the coſine of 45°, the pitch of the roof. And aſſume $y = 2\frac{1}{2}$ inches;

Fig. inches; then $x = 2\frac{1}{2}\sqrt[3]{\frac{81}{3.535}} = 7.1$ inches. And $z = y\sqrt[3]{\frac{ln}{css}} = 2\frac{1}{2}\sqrt[3]{\frac{55}{543}} = 1\frac{1}{6}$ inches.

54. 3. To find the curve ACB, into the form of which, if a joiſt be cut, on the upper or under ſide; and having the two ſides parallel planes, which are perp. to the horizon. That the ſaid joiſt ſhall be equally ſtrong every where to bear a given weight, ſuſpended on it.

Let the weight be placed in the ordinate CD; and the breadth of the beam, and the weight being given; then (Prop. LIV.) the ſtrength at C is as CD^2. And (Prop. LVI.) the ſtreſs is as ADB. Therefore that the ſtrength may be as the ſtreſs, CD^2 is as the rectangle ADB; and therefore the curve ACB is an ellipſis.

55. 4. To find the figure of a beam AB, fixed with one end in a wall, and having a given weight W ſuſpended at the other end B; and being every where of the ſame depth; it may be equally ſtrong throughout.

Let CD be the breadth at C; then (Prop. LIV.) the ſtrength is as CD. And (Cor. 1. Prop. LV.) the ſtreſs is as CB. Therefore CD is every where as CB, and therefore CDB is a plane triangle. And the beam is a priſm, whoſe upper and under ſides are parallel to the horizon.

56. 5. To find the figure of a beam AB, ſticking with one end in a wall, and of a given breadth; having a weight W ſuſpended at the end B; ſo that it may be equally ſtrong throughout.

Let CD be the depth at C. Then ſince the breadth is given, the ſtrength is as CD^2. And the ſtreſs as DB; therefore CD^2 is as DB. Whence CD is a common parabola.

57. 6. To find the figure of a beam AB, of the ſame breadth and depth, ſticking in a wall with

one end, and bearing a weight ſuſpended at the Fig. otherend B; ſo that it may be equally ſtrong throughout. 57.

Let CD be the thickneſs at O. Then the ſtrength is as CD^3, and the ſtreſs is as BO. Therefore BO is as CD^3 or as CO^3. And conſequently ACB is a cubic parabola, whoſe vertex is at B.

7. In like manner, if CBD be a beam fixed with 58. one end in a wall, and all the ſides of it be cut into the form of a concave parabola, whoſe vertex is at B. It will be equally ſtrong throughout for ſupporting its own weight.

For putting $BO = x$, $CO = y$, then by nature of the curve, $ay = xx$. But the ſolidity of CBD is $\frac{3.1416yyx}{5}$. And the center of gravity I, is diſtant $\frac{5}{6}x$ from B, therefore $OI = \frac{1}{6}x$. Now CD^3 or $8y^3 =$ ſtrength at O. And $CBD \times OI$ or $\frac{3.1416yyx}{5} \times \frac{1}{6}x =$ ſtreſs. Therefore the ſtrength : to the ſtreſs : : is as $8y^3 : \text{to } \frac{3.1416y^2xx}{30} :: 8y : \frac{3.1416xx}{30} :: 8y : \frac{3.1416ay}{30} :: 340 : 3.1416a$, that is, in a given ratio. And as this happens every where, the ſolid is equally ſtrong in all parts.

I muſt take notice here that the 116th figure in my large book of Mechanics, is drawn wrong. It ſhould be concave inſtead of being convex.

8. Again, if AB be the ſpire of a church which 59. is a ſolid cone or pyramid; it will be equally ſtrong throughout for reſiſting the wind. For the quantity of wind falling on any part of it ACD, will be as the ſection ACD. Therefore let $AO = x$, $CD = y$. And $x = ay$, then the ſtrength at $O = y^3$, and if I be the center of gravity of ACD, then

OI

Fig. $OI = \frac{1}{3}x$. And the ſtreſs at O = wind ACD ×
59. $OI = xy \times \frac{1}{3}x$. Therefore the ſtrength is to the ſtreſs : : as $y^3 : \frac{1}{3}xxy :: yy : \frac{1}{3}xx$ or $\frac{1}{3}aayy :: 3 : aa$; that is, in a given ratio. Therefore the ſpire is equally ſtrong every where.

SCHOLIUM.

It is all along ſuppoſed that the timber, &c. is of equal goodneſs, where theſe proportions for ſtrength are made. But if it is otherwiſe, a proper allowance muſt be made for the defect.

In theſe Propoſitions, I have called every thing *Strength*, that contributes in a direct proportion to reſiſt any force acting againſt a beam to break it; and I call *Streſs*, whatever weakens it in a direct proportion. But the whole may be referred to the article of ſtrength; for what I have called ſtreſs may be reckoned ſtrength in an inverſe ratio. Thus the ſtrength of a piece of timber may be ſaid to be directly as the breadth and ſquare of the depth, and inverſely as its length, and the weight or force applied; and that is equivalent to taking in the ſtreſs. But I had rather keep them diſtinct, and refer to each of them their proper effects, as I have along done in the foregoing examples.

A piece of wood a foot long, and an inch ſquare, will bear as follows; oak from 320 to 1100; elm from 310 to 930; fir from 280 to 770 pounds, according to the goodneſs.

PROP. LX.

In any machine contrived to raiſe great weights; if the power applied, be to the weight to be raiſed; as the velocity of the weight, to the velocity of the power; the power will only be in equilibrio with the weight. Therefore to raiſe it, the power muſt be ſo far increaſed, as to overcome all the friction and reſiſtance ariſing from the engine or otherwiſe; and then the power will be able to raiſe the weight.

A man would be much miſtaken, who ſhall make an engine to raiſe a great weight, and give his power no greater velocity, in regard to the velocity of the weight; than the quantity of the weight has in regard to the quantity of the power. For when he has done that, his weight and power will but have equal quantities of motion, and therefore they cannot ſet one another a moving, but muſt always remain at reſt. It is neceſſary then, that he do one of theſe two things. 1. That he apply a power greater than in that proportion, ſo much as to overcome all the friction and other accidental reſiſtance that may happen: and in ſome engines theſe are very great. Or 2. He muſt ſo continue his engine, that the velocity of the power, which ſuppoſe he has given, may be ſo much greater than the velocity of the weight; as the quantity of the weight, friction, and reſiſtance and all together, is greater than the power. This being done, the greater power will always overcome the leſſer, and his engine will work.

If a man does not attend to this rule, he will be guilty of many abſurd miſtakes, either in attempting things that are impoſſible, or in not applying means proper for the purpoſe. Hence it is that engines contrived for mines and water-works ſo of-

Fig. ten fail; as they muſt when either the quantity or velocity of the power is too little; or which is the ſame thing, when the velocity of the weight is too great, and therefore would require more power than what is propoſed. As the weight is to move ſlow, the conſequence is, that it will be ſo much a longer time in moving thro' any ſpace. But there is no help for that. For as much as the weight to be raiſed is the greater, the time of raiſing it will be ſo much greater too.

Cor. 1. *Hence in raiſing any weight, what is gained in power is loſt in time. Or the time of riſing thro' any hight will be ſo much longer as the weight is greater.*

If the power be to the weight as 1 to 20, then the ſpace thro' which the weight moves will be 20 times leſs, and the time will be 20 times longer in moving thro' any ſpace, than that of the power. The advantage that is gained by the ſtrength of the motion, is loſt in the ſlowneſs of it. So that tho' they increaſe the power, they prolong the time. And that which one man may do in 20 days, may be done by the ſtrength of twenty men in one day.

Cor. 2. *The quantity of motion in the weight is not at all increaſed by the engine. And if any given quantity of power be immediately applied to a body at liberty, it will produce as much motion in it, as it would do by help of a machine.*

P R O P. LXI.

If an engine be compoſed of ſeveral of the ſimple mechanic powers combined together; it will produce the ſame effect, ſetting aſide friction; as any one ſimple mechanic power would do, which has the ſame power or force of acting.

For let any compound engine be divided into all the ſimple powers that compoſe it. Then the force or power applied to the firſt part, will cauſe it to act upon the ſecond with a new power, which would be deemed the weight, if the machine had no more parts. This new power acting on the ſecond part, will cauſe it to act upon the third part; and that upon a fourth, and ſo on till you come at the weight, which will be acted on, by all theſe mediums, juſt the ſame as by a ſimple machine whoſe power is equal to them all.

Cor. 1. *Hence a compound machine may be made, which ſhall have the ſame power, as any ſingle one propoſed.*

For if a lever is propoſed whoſe power is 100 to 1; two levers acting on one another will be equivalent to it, where the power of the firſt is as 10 to 1, and that of the ſecond alſo as 10 to 1, or the firſt 20 to 1, and the ſecond 5 to 1; or any two numbers, whoſe product is 100.

Again, a wheel and axle whoſe power is as 48 to 1, may be reſolved into two or more wheels with teeth, to have the ſame power; for example, make two wheels, ſo that the firſt wheel and pinion be as 8 to 1, and the ſecond as 6 to 1. They will have the ſame effect as the ſingle one. Or break it into three wheels, whoſe ſeveral powers may be 4 to 1, and 4 to 1, and 3 to 1.

If a ſimple combination of pullies be as 36 to 1; you may take three combinations to act upon one another, whoſe powers are 3 to 1, 3 to 1, and 4 to 1.

And after the ſame manner it is to be done in machines more compounded.

And this is generally done to ſave room. For when an engine is to have great power, it is hardly made of one wheel, it would be ſo large; but by breaking it into ſeveral wheels, after this manner; it will go into a little room, and have the

Fig. ſame power as the other. All the inconvenience is, it will have more friction; for the more parts acting upon one another, the more friction is made.

Cor. 2. *Hence alſo it follows, that in any compound machine, its power is to the weight, in the compound ratio of the power to the weight in all the ſimple machines that compoſe it.*

Cor. 3. *Hence it will be no difficult matter to contrive an engine that ſhall overcome any force or reſiſtance aſſigned.*

For if you have the quantity of power given, as well as of the weight or reſiſtance; it is but taking any ſimple machine as a lever, wheel, &c. ſo that the power may be to the weight in the ratio aſſigned, adding as much to the weight as you judge the friction will amount to. When this ſimple machine is obtained; break it or reſolve it into as many other ſimple ones as you think proper; ſo that they may have the ſame power.

And as to the ſeveral ſimple machines, it matters not what ſort they are of, as to the power; whether they be levers, wheels, pullies, or ſcrews; but ſome are more commodious than others for particular purpoſes; which a mechanic will find out beſt by practice. In general, a lever is the moſt ready and ſimple machine to raiſe a weight a ſmall diſtance; and for further diſtances, the wheel and axle, or a combination of pullies; or the perpetual ſcrew. Alſo theſe may be combined with one another; as a lever with a wheel or a ſcrew, the wheel and axle with pullies, pullies with pullies, and wheels with wheels, the perpetual ſcrew and the wheel. But in general a machine ſhould conſiſt of as few parts as is conſiſtent with the purpoſe it is deſigned for, upon account of leſſening the friction; and to make it ſtill leſs, the joints muſt be oiled or greaſed. All parts that act on one another muſt be poliſhed ſmooth. The axles or ſpindles of wheels muſt not ſhake in the holes, but run true and even. Likewiſe the larger a machine is, if it be well executed, the better and truer it will work. And large wheels and pullies, and ſmall axles or ſpindles have the leaſt friction.

The power applied to work the engine may be men or horſes; or it may be weight or a ſpring; or wind, water, or fire; of which one muſt take that which is moſt convenient and coſts the leaſt. Wind and water are beſt applied to work large engines, and ſuch as muſt be continually kept going. A man may act for a while againſt a reſiſtance of 50 pounds; and for a whole day againſt 30 pounds. A horſe is about as ſtrong as five men.

If two men work at a roller, the handles ought to be at right angles to one another.

When a machine is to go regular and uniform, a heavy wheel or fly muſt be applied to it.

SCHOLIUM.

Two things are required to make a good engineer. 1. A good invention for the ſimple and eaſy contrivance of a machine, and this is to be attained by practice and experience. 2. So much theory as to be able to compute the effect any engine will have; and this is to be learned from the principles of Mechanics.

PROP. LXII.

The friction or reſiſtance ariſing by a body moving upon any ſurface, is as the roughneſs of the ſurface, and nearly as the weight of the body; but is not much increaſed by the quantity of the ſurface of the moving body, and is ſomething greater with a greater velocity.

It is matter of experience that bodies meet with a great deal of reſiſtance by ſliding upon one another, Fig. ther, which cannot be entirely taken away, tho' the bodies be made never ſo ſmooth : yet by ſmoothing or poliſhing their ſurfaces, and taking off the roughneſs of them, this reſiſtance may be reduced to a ſmall matter. But many bodies, by their natural texture, are not capable of bearing a poliſh ; and theſe will always have a conſiderable degree of reſiſtance or friction. And thoſe that can be poliſhed, will have ſome of this reſiſtance ariſing from the coheſion of their ſurfaces. But in general, the ſmoother or finer their ſurfaces, the leſs the friction will be.

As the ſurfaces of all bodies are in ſome degree rough and uneven, and ſubject to many inequalities ; when one body is laid upon another, the prominent parts of one fall into the hollows of the other ; ſo that the body cannot be moved forward, till the prominent parts of one be raiſed above the prominent parts of the other, which requires the more force to effect, as theſe parts are higher ; that is, as the body is rougher. And this is ſimilar to drawing a body up an inclined plane, for theſe protuberances are nothing elſe but ſo many inclined planes, over which the body is to be drawn. And therefore the heavier the body, the more force is required to draw it over theſe eminencies ; whence the friction will be nearly as the weight of the body.

But whilſt the roughneſs remains the ſame, or the prominent parts remain of the ſame hight, there will always be required the ſame force, to draw the ſame weight. And the increaſing of the ſurface, retaining the ſame weight, can add nothing to the reſiſtance on that account ; but it will make ſome addition upon other accounts. For when one ſurface is dragged along another, ſome part of the reſiſtance ariſes from ſome parts of the moving ſurface, taking hold of the parts of the other, and tearing them off ; and this is called *wearing*. And therefore this part of the friction is greater in a greater ſurface, in proportion to that ſurface. There is likewiſe in a greater ſurface, a greater force of coheſion, which ſtill adds ſomething to the friction. But the two parts of the friction, ariſing from the wearing and tenacity, are not increaſed by the velocity: but the other part, of drawing them over inclined planes, will increaſe with the velocity. So that in the whole, the friction is ſomething increaſed by the quantity of the ſurface, and by the velocity, but not much. But more in ſome bodies than others, according to their particular texture. Fig.

Cor. 1. *Hence there can be no certain rule, to eſtimate the friction of bodies; this is a matter that can only be decided by experiments. But it may be obſerved, that, ceteris paribus, hard bodies will have leſs reſiſtance than ſofter; and bodies oiled or greaſed, will have far leſs.*

For the particles of hard bodies, cannot ſo well take hold of one another to tear themſelves off. And when a ſurface is oiled, it is the ſame thing as if it run upon a great number of rollers or ſpheres.

Cor. 2. *Hence alſo a method appears of meaſuring the friction of a body ſliding upon another body, by help of an inclined plane.*

Take a plank CB of the ſame matter, raiſe it at one end C ſo high, till the body whoſe friction is ſought, being laid at C, ſhall juſt begin to move down the plane CB. Then the weight of the body, is to the friction as the baſe AB, to the hight AC of the plane. For the preſſure againſt the plane is the part of the weight that cauſes the friction, and the tendency down the plane is equal to the friction. And (Prop. XIV.) that preſſure is to the tendency as AB to AC. 60.

Fig. 60. If you puſh the body from C downward, and obſerve it to keep the ſame velocity thro' D to B; then you will have the friction for that velocity. If it increaſes its velocity, lower the end of the plank C; if it grows ſlower, raiſe the end C, till you get the body to have the ſame velocity quite thro' the plane. And ſo you will find what elevations are proper for each velocity; and from thence the ratio of AB to AC, or of the weight to the friction.

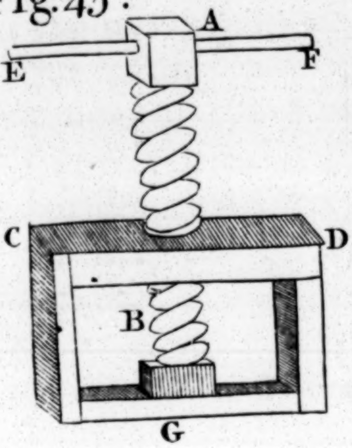
There is a way to make the experiment, by drawing the body along a horizontal plane, by weights hung at a ſtring, which goes over a pulley; but the method here deſcribed is more eaſy and ſimple.

SCHOLIUM.

From what has been before laid down, it will be eaſy to underſtand the nature of engines, and how to contrive one for any purpoſe aſſigned. And likewiſe having any engine before us, we can by the ſame rules, compute its powers and operations.

Engines are of various kinds; ſome are fixed in a particular place, where they are to act; as wind-mills and water-mills for corn, fire engines for drawing water, gins for coal pits, many ſorts of mills; pumps, cranes, &c. others are movable from one place to another, and may be carried to any place where they are wanted, as blocks, pullies and tackles for raiſing weights, the lifting jack, and lifting ſtock, clocks, watches, ſmall bellows, ſcales, ſteelyards, and an infinite number of others. Another ſort of engines are ſuch as are made on purpoſe to move from one place to another, ſuch as boats, ſhips, coaches, carriages, waggons, &c. If any of theſe are urged forward by the help of levers, wheels, &c. By having the acting power given, the moving force that drives it forward, is eaſily found by the properties of theſe machines. Only obſerve, if the firſt acting power

Fig. 45.



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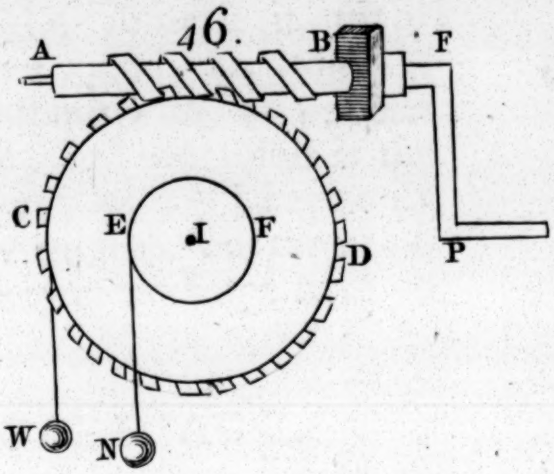

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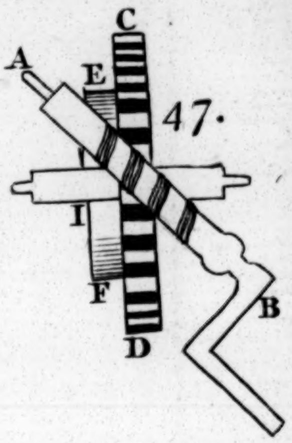

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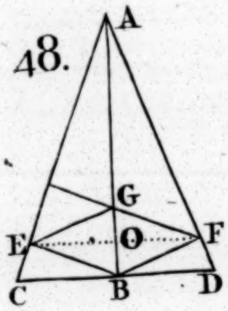

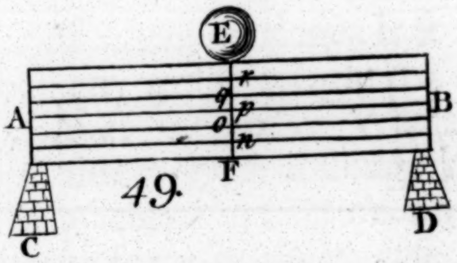

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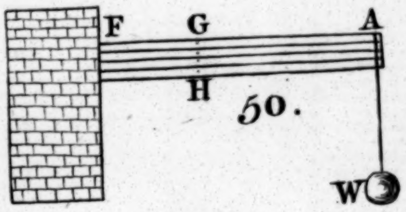

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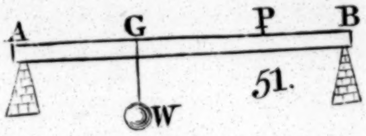

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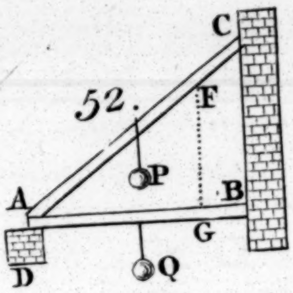

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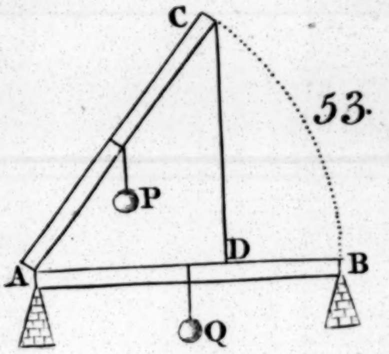

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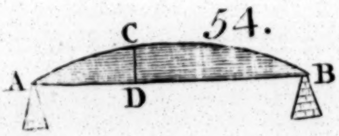

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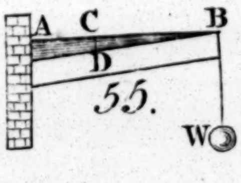

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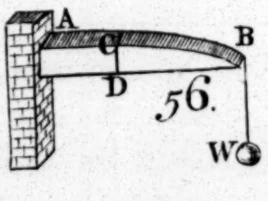

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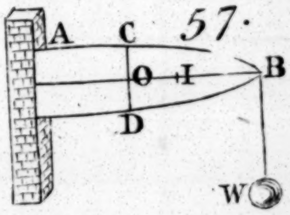

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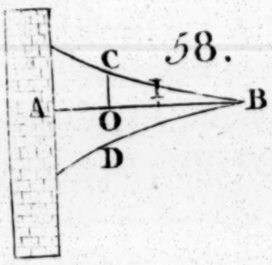

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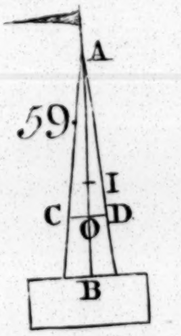

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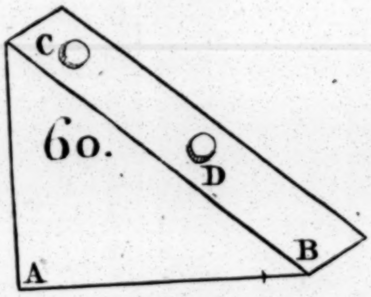


Pl. IV. pa. 102.

Mechanics

power be external, as wind, water, horſes, &c. Fig.
you muſt not forget to add or ſubtract it, to or from the moving force before found; according as that firſt acting power conſpires with, or oppoſes the motion of the machine; and the reſult is the true force it is driven forward with. I have only room to deſcribe a very few engines, but thoſe that deſire it may ſee great variety in my large book of Mechanics.

A WHEEL CARRIAGE.

61. AB is a cart or carriage, going upon two wheels as CD, and ſometimes upon four, as all waggons do. The advantages of wheel carriages is ſo great, that no body who has any great weight to carry, will make uſe of any other method. Was a great weight to be dragged along upon a ſledge or any ſuch machine without wheels, the friction would be ſo great, that a ſufficient force in many caſes could not be got to do it. But by applying wheels to carriages, the friction is almoſt all of it taken away. And this is occaſioned by the wheels turning round upon the ground, inſtead of dragging upon it. And the reaſon of the wheel's turning round is the reſiſtance the earth makes againſt it at O where it touches. For as the carriages goes along, the wheel meets with a reſiſtance at the bottom O, where it touches the ground; and meeting with none at the top at C, to balance it; that force at O muſt make it turn round in the order ODC, ſo that all the parts of the circumference of the wheel are ſucceſſively applied to the earth. In going down a ſteep bank it is often neceſſary to tie one wheel faſt, that it cannot turn round, this will make it drag; and by the great reſiſtance it meets with, ſtops the too violent motion, the carriage would otherwiſe have, in deſcending the hill.

Fig. 61. But altho' all ſorts of wheels very much diminiſh the friction; yet ſome have more than others; and it may be obſerved that great wheels, and ſmall axles have the leaſt friction. To make the friction as little as poſſible, ſome have applied friction wheels, which is thus; EG is the friction wheel running upon an axis I which is fixed in the piece of timber ES, which timber is fixed to the ſide of the carriage. KL is the axle of the carriage, which is fixed in the wheel CD, ſo that both turn round together. Then inſtead of the carriage lying upon the axle KL, the friction wheel FG lies upon the axle; ſo that when the wheel CD turns round, the axle cauſes the friction wheel, with the weight of the carriage upon it, to turn round the center I, which diminiſhes the friction in proportion to the radius IG: and there is the ſame contrivance for the wheel on the other ſide. But the wheel CD need not be fixed to the axle; for it may turn round on the axle KL, and alſo the axle turn round under the carriage.

In paſſing over any obſtacles, the large wheels have the advantage. For let MN be an obſtacle; then drawing the wheel over this obſtacle, is the ſame thing as drawing it up the inclined plane MP, which is a tangent to the point M; but the greater the wheel CD is, the leſs is that plane inclined to the horizon.

Likewiſe great wheels do not ſink ſo deep into the earth as ſmall ones, and conſequently require leſs force to pull them out again.

But there are diſadvantges in great wheels; for in the firſt place, they are more eaſily overturned; and ſecondly, they are not ſo eaſy to turn with, in a ſtrait road as ſmall wheels.

The tackle of any carriage ought to be ſo fixed, that the horſe may pull partly upwards, or lift, as well as pull forwards; for all hills and inequalities in the road, being like ſo many inclined planes, Fig. the weight is moſt eaſily drawn over them, when 61. the power draws at an equal elevation.

A carriage with four wheels is more advantageous, than one with two only, but they are bad to turn; and therefore are obliged to make uſe of ſmall fore-wheels. Broad wheels which are lately come into faſhion, are very advantageous, as they ſink but little into the earth. But there is a diſadvantage attends them, for they take up ſuch a quantity of dirt by their great breadth, as ſenſibly retards the carriage by its weight, and the like may be ſaid of their own weight.

The under ſide of the axle where the wheels are, muſt be in a right line; otherwiſe if they ſlant upwards, the weight of the carriage will cauſe them to work toward the end, and preſs againſt the runners and lin pin. And as the ends of the axle are conical, this cauſes the wheels to come nearer together at bottom, and be further diſtant at the top; by which means the carriage is ſooner overturned. To help this, the ends of the axle muſt be made as near a cylindrical form as poſſible, to get the wheels to fit, and to move free.

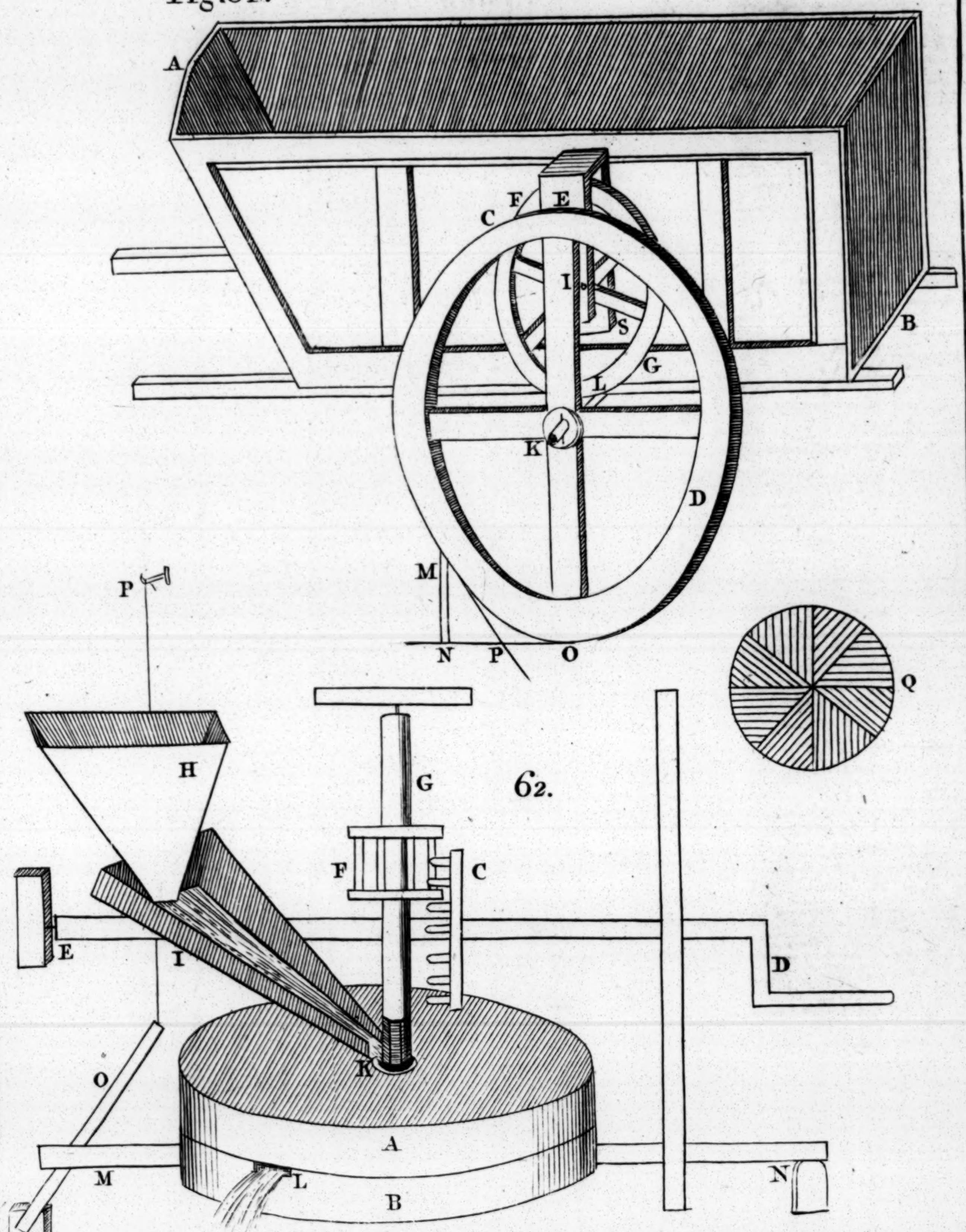
A HAND MILL.

Fig. 62. is a hand mill for grinding corn, A, B 62. the ſtones included in a wooden caſe. A the upper ſtone, being the living or moving ſtone. B the lower ſtone, or the dead ſtone, being fixed immovable. The upper ſtone is 5 inches thick, and a foot and three quarters broad; the lower ſtone is broader. C is a cog-wheel, with 16 or 18 cogs; DE its axis. F is a trundle with 9 rounds, fixed to the axis G, which axis is fixed to the upper ſtone A, by a piece of iron made on purpoſe. H is the hopper, into which the corn is put; I the ſhoe, to carry the corn by little and little thro' a hole at K,

Fig. 62. K, to fall between the two ſtones. L is the mill eye, being the place where the flour or meal comes out after it is ground. The under ſtone is ſupported by ſtrong beams not drawn here. And the ſpindle G ſtands on the beam MN, which lies upon the bearer O, and O lies upon a fixed beam at one end, and at the other end has a ſtring fixed, and tied to the pin P. The under ſtone is not flat, but riſes a little in the middle, and the upper one is a little hollow. The ſtones very near touch at the out ſide, but are wider towards the middle to let the corn go in.

When corn is to be ground, it is put into the hopper H, a little at a time, and a man turns the handle D, which carries round the cog-wheel C, and this carries about the trundle F, and axis G, and ſtone A. The axis G is angular at K; and as it goes round, it ſhakes the ſhoe I, and makes the corn fall gradually thro' the hole K. And the upper ſtone going round grinds it, and when ground it comes out at the mill eye L, where there is a ſack or tub placed to receive it. Another handle may be made at E like that at D, for two men to work, if any one pleaſes. In order to make the mill grind courſer or finer, the upper ſtone A may be lowered or raiſed, by means of the ſtring going from the bearer O; for turning round the pin P, the ſtring is lengthened or ſhortened, and thereby the timbers O, M are lowered or raiſed, and with them the axle G and ſtone A. For the ſpindle G goes thro' the ſtone B, and runs upon the beam MN. The ſpindle is made ſo cloſe and tight, by wood or leather, where it goes thro' the under ſtone, that no meal can fall thro'. The under ſide of the upper ſtone is cut into gutters in the manner repreſented at Q. It is a pity ſome ſuch like mills are not made at a cheap rate for the ſake of the poor, who are much diſtreſſed by the roguery of the millers.

Fig. 61.



Mechanics

Pl. V. *pa.* 106.

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Fig. 63. is a ſort of crane, BC an upright poſt, AB a beam fixed horizontally at top of it; theſe turn round together on the pivot C, and within the circle S, which is fixed to the top of the frame PQ. EF is a wooden roller, or rather a roller made of thin boards, for lightneſs, and all nailed to ſeveral circular pieces on the inſide. GH a wheel fixed to the roller, about which goes the rope GR. IK, LN, two other ropes; fixed with one end to the croſs piece AB, and the other end to the roller EF. W a weight equal to the weight of the wheel and roller, which is faſtened to a rope which goes over the pulley O, and then is faſtened to a collar V, which goes round the roller. ET is another rope with a hook at it to lift up any weight, the other end of the rope being fixed to the roller; here are in all five ropes. Fig. 63.

To raiſe any weight as M, hang it upon the hook T, then pulling at the rope R which goes about the wheel GH, this cauſes the wheel and roller to turn round, and the ropes IK, LN to wind about it, by which means the wheel and axle riſes; and by riſing, folds the rope TE about the roller the contrary way, and ſo raiſes the weight M. When the weight M is raiſed high enough, a man muſt take hold of the rope T with a hook, by which the whole machine may be drawn about, turning upon the centers C and S. And then the weight M may be let down again. The weight of the wheel and roller do not affect the power drawing at R, becauſe it is balanced by the weight W. There is no friction in this machine but what is occaſioned by the collar V, and the bending of the ropes. And the power is to the weight in this crane, as the diameter of the roller to the radius of the wheel GH.

Fig.

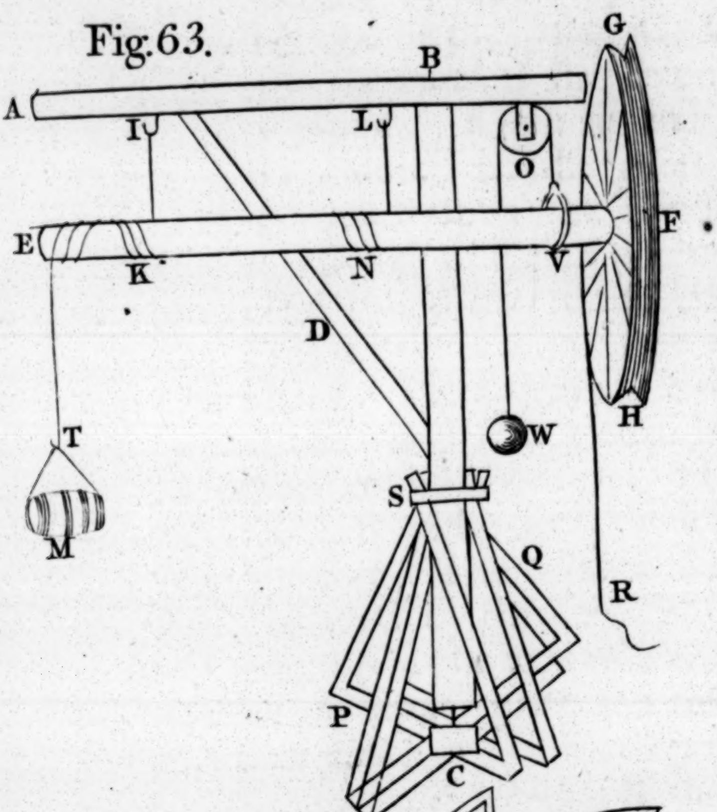
An ENGINE for raiſing WEIGHTS.

64. Fig. 64. is an engine compoſed of a perpetual ſcrew AB, and a wheel DE with teeth, and a ſingle pulley H. FG is an axle, about which a rope goes, which lifts the pulley and weight W. BC is the winch, to turn it round withal. As the ſpindle AB is turned about, the teeth of it takes the teeth of the wheel DE, and turns it about, together with the axle FG, which winds up the rope, and raiſes the pulley H, with the weight W. The power at C, is to the weight W, as diameter FG × by the breadth of one tooth, is to twice the diameter DE × circumference of the circle deſcribed by C.

A FULLING MILL.

65. Fig. 65. is a fulling mill. AB a great water wheel, carried about by a ſtream of water, coming from the trough C, and falling into the buckets D, D, D whoſe weight carries the wheel about; this is a breaſt mill, becauſe the water comes no higher than the middle or breaſt of the wheel; EF is its axis; I, I; K, K, two lifters going thro' the axle, which raiſe the ends G, G of the wooden mallets GH, GH, as the wheel goes about; and when the end G ſlips off the cog or lifter K or I, the mallet falls into the trough L, and each of the mallets makes two ſtrokes for one revolution of the wheel. The mallets move about the centers M, M. Theſe troughs L, L, contain the ſtuff which is to be milled, by the beating of the mallets. N, N, is a channel to carry the water, being juſt wide enough to let the wheel go round. And the wheel may be ſtopt, by turning the trough C aſide, which brings the water. In this engine more mallets may be uſed, and then more pins or lifters muſt be put thro' the axis EF.

Fig. 63.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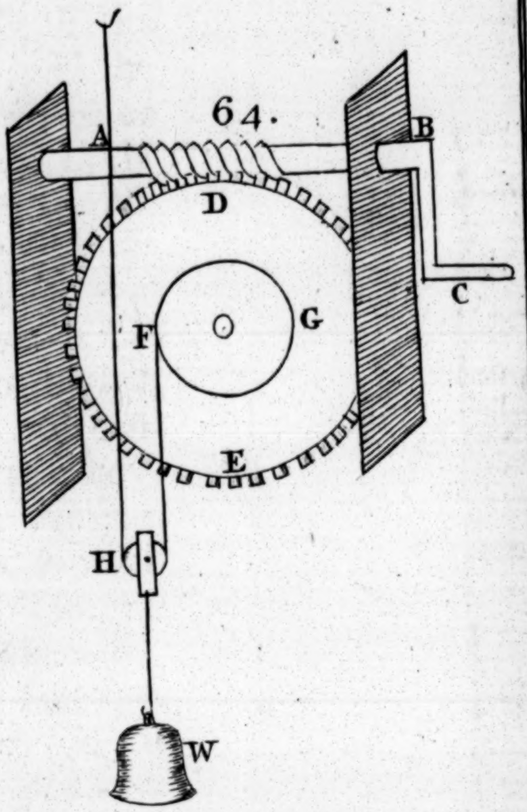
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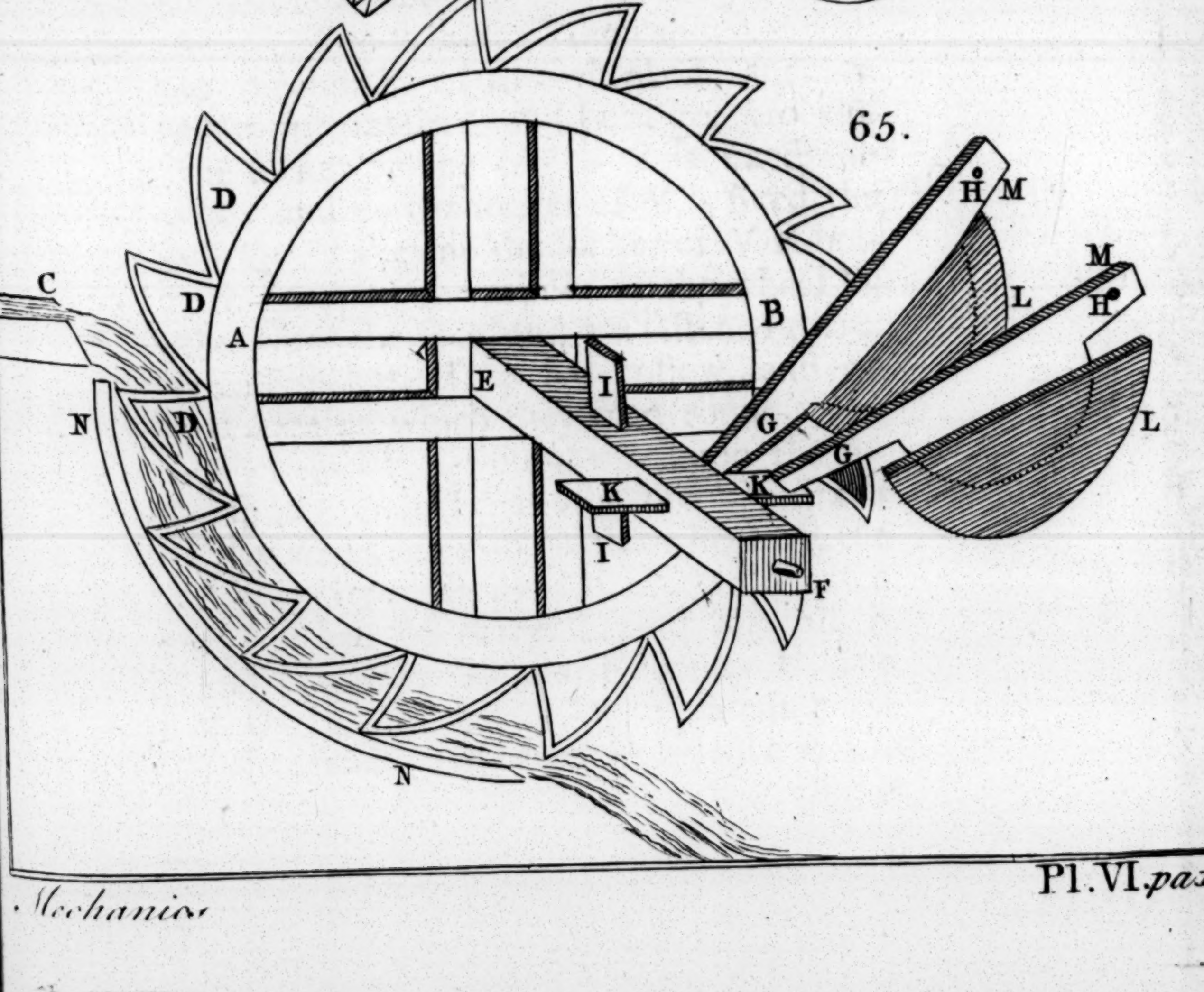
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Fig. 66. is a common *Pocket Watch.* AA the *balance,* BB the *verge*; C, C, two *palats.* D the *crown wheel* acting againſt the palats C, C; E its pinion. F the *contrate wheel,* G its pinion. H the *third wheel,* I its pinion. K the ſecond wheel or *center wheel,* L its pinion. M the *great wheel,* N the *fuſee* turning round upon the ſpindle of M. O the *ſpring box,* having a ſpring included in it. PP the *chain* going round the ſpring box O, and the fuſee N. This work is within the watch between the two plates. Here the face is downward, and in the watch the wheel K is placed in the center, and the others round about it. Here I have placed them ſo as beſt to be ſeen, which ſignifies nothing to the motion. The balance AA is without the plate, covered by the cock X. The minute hand Q goes upon the axis of the wheel K. Fig. 66.

Then between the upper plate and the face, we have V the *cannon pinion* or *pinion of report.* Z the *dial wheel.* T the *minute wheel.* S the *pinion* or *nut,* fixed to it. The ſocket of the cannon pinion V goes into the ſocket of the wheel Z, and are movable about one another, and both go thro' the face; on the ſocket of the pinion Z is fixed the hour hand R; and on the ſocket of V is fixed the minute hand Q. Likewiſe the ſocket of V is hollow, and both go upon the arbour of the wheel K, which reaches thro' the face, and are faſtened there. The wheel and ſocket, T, S are hollow, and go upon a fixed axle on which they turn round.

When the chain PP is wound up, upon the fuſee N; the ſpring included in the box O, draws the chain PP, which forces about the wheel M, the fuſee being kept from ſlipping back, by a catch on purpoſe. Then M drives L and K, and K drives I, and H drives G, and F drives E, and the teeth of the crown wheel D, act againſt the palats C, C alternately, and cauſe the balance AA to vibrate

Fig. vibrate back and forward, and thus the watch is kept going.

66. The cannon pinion, and dial wheel V and Z, and the hands Q, R, being put upon the arbor of K at W; and faſtened there, by means of a ſhoulder which is upon the axis, and a braſs ſpring; as the wheel K goes round, it carries with it the pinion V with the minute hand, and V drives T together with S; and S drives Z with the hour hand.

The numbers of the wheels and pinions, (that is the teeth in them) are, M = 48, L = 12, K = 54, I = 6, H = 48, G = 6, F = 48, E = 6, D = 15, and 2 palats. The *train*, or number of beats in an hour, is 17280, which is about $4\frac{3}{4}$ beats in a ſecond. Alſo V = 10, Z = 36, S = 12, T = 40.

The wheel M goes round 6 times in 24 hours, therefore K goes round $\left(\frac{48}{12}\right)$ 4 times as much; that is, 24 times, or once in an hour, and the hand Q along with it; therefore Q will ſhew minutes. Then as V goes round once in an hour, T will go round $\left(\frac{10}{40}\right)$ $\frac{1}{4}$ of that, or $\frac{1}{4}$ the circumference; and as S goes $\frac{1}{4}$, Z will go $\left(\frac{12}{36}\right)$ $\frac{1}{3}$ of that, or $\frac{1}{12}$ of the circumference in an hour, and therefore as R goes along with it, R will ſhew the hours. The wheels and pinions T, Z, and S, V, are drawn with the face upwards. And the whole machine included in a caſe is but about two inches diameter. There is a ſpiral ſpring fixed under the balance AB, called the *regulator*, which gives it a regular motion; and likewiſe abundance of ſmall parts helpful to her motion, too long to be deſcribed here.

The way of writing down the numbers, is thus, Fig. 66.

```
48
12 — 54                          10 Q
      6 — 48                     40 — 12
            6 — 48                     36 R.
                  6 — 15
                        2
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Explanation. The wheel with 48 drives a pinion of 12, and a wheel of 54 on the ſame arbor. The wheel 54 drives the pinion 6 with the wheel 48 on the ſame arbor. The wheel 48 drives the pinion 6 and wheel 48 on the ſame arbor. The wheel 48 drives the pinion 6 and wheel 15 on the ſame arbor. And the wheel 15 drives the two palats.

Again the wheel 54 has the pinion 10 on its arbor, and the hand Q; and the pinion 10 drives the wheel 40, with the pinion 12. And the pinion 12 drives the wheel 36 with the hand R.

As this machine is moved by a ſpring, it is ſubject to very great inequalities of motion, occaſioned by heat and cold. For hot weather ſo relaxes, ſoftens, and weakens the main ſpring, that it loſes a great deal of its ſtrength, which cauſes the watch to loſe time and go too ſlow. On the other hand, cold froſty weather ſo affects the ſpring, and it is ſo condenſed and hardened, that it becomes far ſtronger; and by that means accelerates the motion of the watch, and makes her go faſter. The difference of motion in a watch, thus occaſioned by heat and cold, will often amount to an hour, and more in 24 hours. To remedy this, there is a piece of machinery, called the *Slide*, placed near the regulating ſpring; which being put forward or backward, ſhortens or lengthens the ſpring, ſo as to make her keep time truly.

Some people have been ſo ſilly as to think, that the greater ſtrength of a ſpring ariſes wholly from

Fig. 67. from its being made ſhorter, as this happens to be one of the effects of cold. But it is eaſily demonſtrated that this is not the cauſe. For let AB be a ſpring as it is dilated by heat, and *ab* the ſame ſpring contracted by cold. Now if the ſpring has been contracted in length, it muſt be proportionally contracted in all dimenſions. Let *l*, *b*, *d*, denote the length, breadth, and depth, in its cold, and leaſt dimenſions; and *rl*, *rb*, *rd*, the length, breadth, and depth, in its hot and greateſt dimenſions. Then (Prop. LIV.) the ſtrength of the longer, to the ſtrength of the ſhorter, will be as $\frac{rb \times rrdd}{rl}$ to $\frac{bdd}{l}$ (conſidering it weakened by the length), and that is as rr to 1, or as AB^2 to ab^2. So that the longer ſpring, upon account of its being affected with heat, is ſo far from being weaker, than the ſhorter affected with cold, that it is the ſtronger of the two. And therefore this difference is not to be aſcribed merely to the lengthning or ſhortning thereof; but muſt be owing to the nature, texture and conſtitution of the ſteel, as it is ſome way or other affected and changed by the heat and cold.

And that there is ſome change induced by the cold, into the very texture of the mettal, is evident from this, that all ſorts of tools made of iron or ſteel, as ſprings, knives, ſaws, nails, &c. very eaſily ſnap and break in cold froſty weather, which they will not do in hot weather. And that property of ſteel ſprings is the true cauſe, that theſe ſorts of movements can never go true.

66. To make a calculation of the different forces requiſite to make a watch gain or loſe any number of minutes, as ſuppoſe half an hour in 24; and I have often experienced it to be more. By Cor. 4. Prop. VI. the product of the force and ſquare of the

Fig. 66.

the time, is as the product of the body and ſpace deſcribed, which here is a given quantity. For the matter of the balance remains the ſame in hot as cold weather; and ſo does the length of the ſwing, which here is the ſpace deſcribed. Therefore the force is reciprocally as the ſquare of the time of vibrating, or directly as the ſquare of the number of vibrations in 24 hours. Therefore the force with the warm ſpring, is to the force with the cold one; as the ſquare of 23 $\frac{1}{2}$ hours, to the ſquare of 24; that is, nearly as 23 to 24. So that if a ſpring was to contract half an inch in a foot in length, without altering its other dimenſions, it would but be ſufficient to account for that phænomenon; but this is forty times more than the lengthening and ſhortening by heat and cold, for that does not alter ſo much as a thouſandth part, as is plain from experiments.

The caſe being thus, a clock or watch going by a ſpring, can never be made to keep time truly, except it be always kept to the ſame degree of heat or cold, which cannot be done without conſtant attendance. And if any ſort of mechaniſm be contrived to correct this; yet as ſuch a thing can only be made by gueſs, it cannot be truſted to at ſea, but only for ſhort voyages. But no motion however regular, can ever anſwer at ſea, where the irregular motion of the ſhip will continually diſturb it; add to this, that the ſmall compaſs a watch is contained in, makes it eaſier diſturbed, than a larger machine would be; but to ſuppoſe that any regular motion can ſubſiſt among ten thouſand irregular motions, and in ten thouſand different directions, is a moſt glaring abſurdity. And if any one with ſuch a machine would but make trial of it to the Eaſt Indies, he would find the abſurdity and diſappointment. And therefore I never expect to ſee ſuch a time keeper, or any ſuch thing as a watch or clock

Fig. 66. going by a ſpring, to keep true time at ſea. But time will diſcover all things.

As to pendulum clocks, their irregularity in the ſame latitude is owing to nothing but the lengthning or ſhortning of the pendulum; which is a mere trifle to the other. But then they would be infinitely more diſturbed at ſea, than a watch; and in a ſtorm could not go at all. In different latitudes too, another irregularity attends a pendulum, depending on the different forces of gravity. Tho' this amounts but to a ſmall matter, yet it makes a conſiderable variation, in a great length of time. For in ſouth latitudes, where the gravity is leſs, a clock loſes time. And in north latitudes, where the gravity is greater, it gains time. So that none of theſe machines are fit to meaſure time at ſea, altho' ten times ten thouſand pounds ſhould be given away for making them.

A DESCENDING CLOCK.

68. Fig. 68. is a clock deſcending down an inclined plane. This conſiſts of a train of watch work, contained between two circular plates AB, CD, 4 inches diameter, fixed together by a hoop an inch and half broad, incloſing all the work. The inner work conſiſts of 5 wheels, the ſame as in a watch, only there is a ſpur wheel inſtead of the contrate wheel, as 4; *b* is the balance, whoſe palats play in the teeth of the crown wheel 5. Here is no ſpring to give it motion, but inſtead thereof, the weight W is fixed to the wheel 1, and ſo adjuſted for weight, that it may balance the lower ſide, and hinder it from rolling down the plane. Now whilſt the weight W moves the wheel 1, this wheel by moving about, cauſes the weight W to deſcend, by which it ceaſes to be a balance for the oppoſite ſide, and therefore that ſide begins to deſcend,

ſcend, till the weight W be raiſed high enough again to become a balance, which muſt be about the poſition it appears in the figure. Thus whilſt wheels move gradually about, the weight W deſcends gradually, which makes the body of the machine turn gradually round, and deſcend down the inclined plane PQ; making one revolution in 12 hours. And therefore to have her to go 24 or 30 hours; the length of the plane PQ muſt be 2 or 2 $\frac{1}{2}$ circumferences of the plates. Before the weight W is fixed to the wheel 1, ſome lead or braſs muſt be ſoldered on the ſide E oppoſite to the wheels 2, 3, 4, &c. for the wheel 1 muſt be in the center. And then the lead or braſs muſt be filed away till the center of gravity of the machine be in the center of the plates. And to hinder the machine from ſliding, the edges of the plates muſt be lightly indented. The inclined plane PQ may be a board, which muſt be elevated 10 or 12 degrees, but that is to be found by trials; for if ſhe go too ſlow the end P muſt be raiſed; but if too faſt it muſt be lowered. When the clock has gone the length of the board to Q, it muſt be ſet again at P. The fore ſide CD is divided into hours, and a pin is fixed in the center at G, on which the hand FGH, always hangs looſely in a perp. poſition, with the heavy end H downward. And the end F ſhews the hour of the day. So that the hours come to the hand, and not the hand to the hours.

Fig. 68.

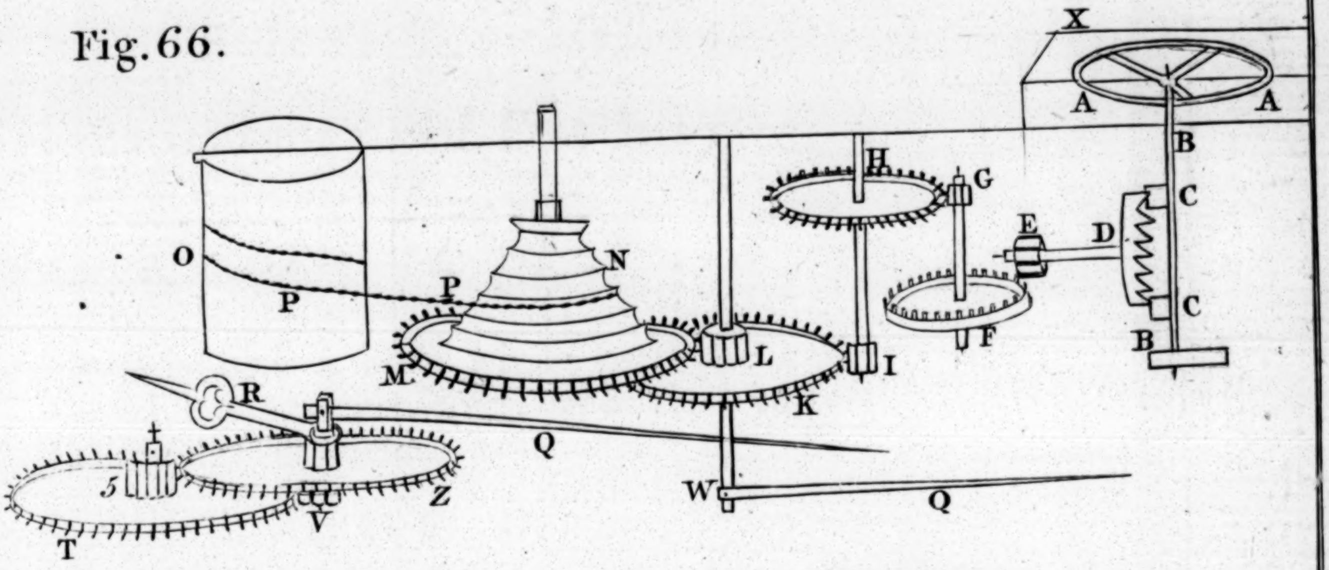
The board PQ muſt be be perfectly ſtreight from one end to the other, or elſe ſhe will go faſter in ſome places, and ſlower in others.

The circle with hours ought to be a narrow rim of braſs, movable round about, by the help of of one or more pins placed in it; ſo that it may be ſet to the true time.

Fig. 68. The weight W ſerves for two uſes, 1, to be a counterpoiſe to the ſide A; and 2, by its weight to put the clock in motion.

The weight W muſt be ſo heavy as to make the clock keep time, when it has a proper degree of elevation as 45 degrees; and then the board muſt have an elevation of 10 or 12 degrees. If ſhe go too faſt, with theſe poſitions, take ſome thing off the weight; if too ſlow, add ſomething to it.

Fig. 66.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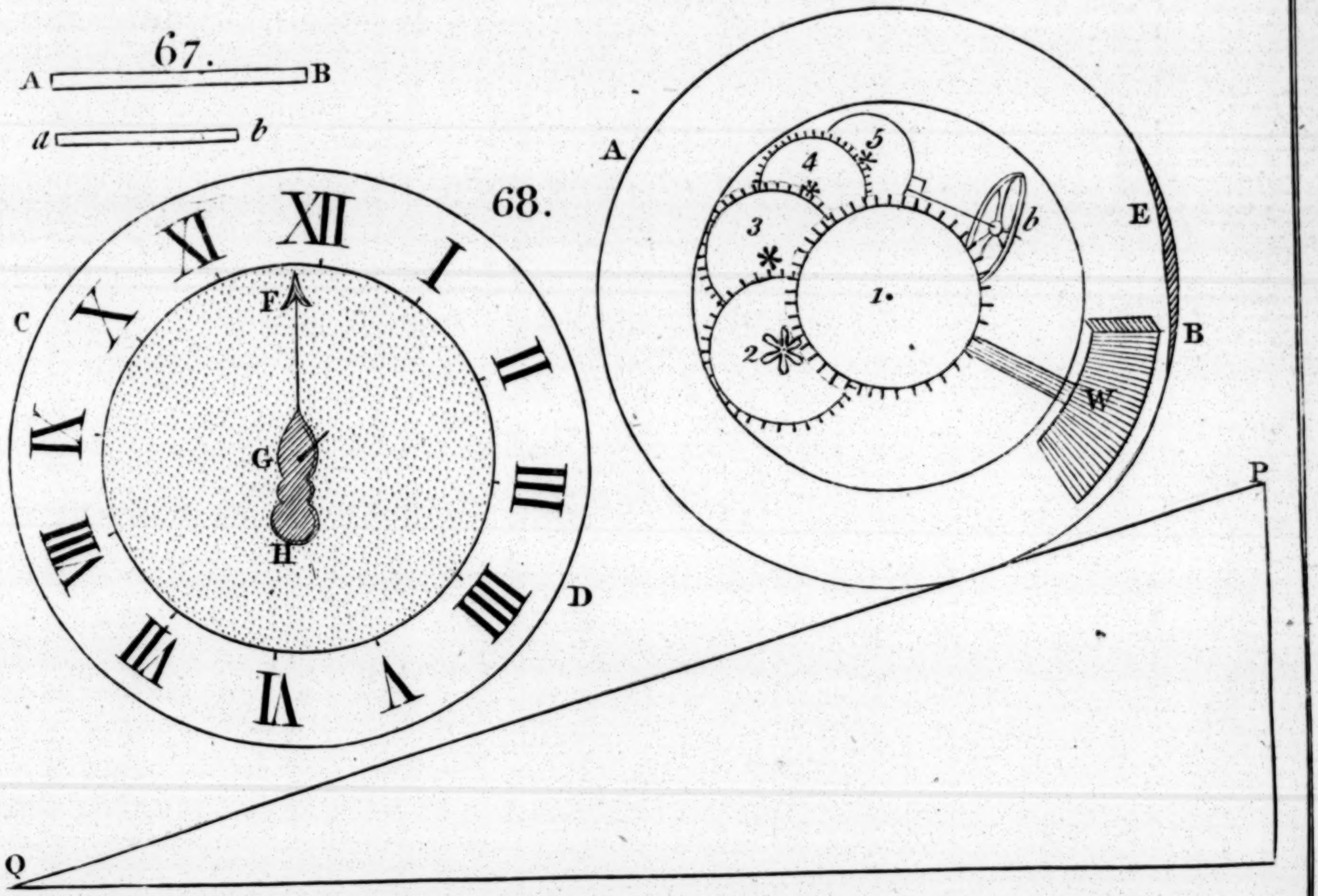
67.

68.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SECT. VI.

HYDROSTATICS *and* PNEUMATICS.

DEFINITION I.

A Fluid, is ſuch a body whoſe parts are eaſily moved among themſelves, and yield to any force acting againſt them.

DEF. II.

Hydroſtatics, is a ſcience that demonſtrates the properties of fluids.

DEF. III.

Hydraulics, is the art of raiſing water by engines.

DEF. IV.

Pneumatics, is that ſcience which ſhews the properties of the air.

DEF. V.

A fountain or *jet d'eau*, is an artificial ſpout of water.

PROP. LXIII.

If one part of a fluid be higher than another, the higher parts will continually deſcend to the lower places, and will not be at reſt, till the ſurface of it is quite level.

For the parts of a fluid being movable every way, if any part is above the reſt, it will deſcend by its own gravity as low as it can get. And afterwards other parts that are now become higher, will

Fig. will deſcend as the other did, till at laſt they will all be reduced to a level or horizontal plane.

Cor. 1. *Hence water that communicates by means of a channel or pipe, with other water; will ſettle at the ſame hight in both places.*

Cor. 2. *For the ſame reaſon, if a fluid gravitates towards a center; it will diſpoſe itſelf into a ſpherical figure, whoſe center is the center of force. As the ſea in reſpect of the earth.*

PROP. LXIV.

If a fluid be at reſt in a veſſel whoſe baſe is parallel to the horizon; equal parts of the baſe are equally preſſed by the fluid.

For upon every part of the baſe there is an equal column of the fluid ſupported by it. And as all theſe columns are of equal weight, they muſt preſs the baſe equally; or equal parts of the baſe will ſuſtain an equal preſſure.

Cor. 1. *All parts of the fluid preſs equally at the ſame depth.*

For imagine a plain drawn thro' the fluid parallel to the horizon. Then the preſſure will be the ſame in any part of that plane, and therefore the parts of the fluid at the ſame depth ſuſtain the ſame preſſure.

Cor. 2. *The preſſure of a fluid at any depth, is as the depth of the fluid.*

For the preſſure is as the weight, and the weight is as the hight of a column of the fluid.

PROP. LXV.

If a fluid is compreſſed by its weight or otherwiſe; at any point it preſſes equally, in all manner of directions.

This ariſes from the nature of fluidity; which is, to yield to any force in any direction. If it cannot give way to any force applied, it will preſs againſt other parts of the fluid in direction of that force. And the preſſure in all directions with be the ſame. For if any one was leſs, the fluid would move that way, till the preſſure be equal every way.

Cor. *In any veſſel containing a fluid; the preſſure is the ſame againſt the bottom, as againſt the ſides, or even upwards, at the ſame depth.*

PROP. LXVI.

The preſſure of a fluid upon the baſe of the containing veſſel, is as the baſe, and perpendicular altitude; whatever be the figure of the veſſel that contains it. 69.

Let ABIC, EGKH be two veſſels. Then (Prop. LXIV. Cor. 2.) the preſſure upon an inch on the baſe AB = hight CD × 1 inch. And the preſſure upon an inch on the baſe HK is = hight FH × 1 inch. But (Prop. LXIV.) equal parts of the baſes are equally preſſed, therefore the preſſure on the baſe AB is CD × number of inches in AB; and preſſure on the baſe HK is FH × number of inches in HK. That is, the preſſure on AB is to the preſſure on HK; as baſe AB × hight CD, to the baſe HK × hight FH.

Cor. 1. *Hence if the hights be equal, the preſſures are as the baſes. And if both the hights and baſes be equal;*

Fig. 69. *equal; the preſſures are equal in both; tho' their contents be never ſo different.*

For the reaſon that the wider veſſel EK, has no greater preſſure at the bottom, is, becauſe the oblique ſides EH, GK, take off part of the weight. And in the narrower veſſel CB, the ſides CA, IB, re-act againſt the preſſure of the water, which is all alike at the ſame depth; and by this re-action the preſſure is increaſed at the bottom, ſo as to become the ſame every where.

Cor. 2. *The preſſure againſt the baſe of any veſſel, is the ſame as of a cylinder of an equal baſe and hight.*

70. Cor. 3. *If there be a recurve tube* ABF, *in which are two different fluids* CD, EF. *Their hights in the two legs* CD, EF, *will be reciprocally as their ſpecific gravities, when they are at reſt.*

For if the fluid EF be twice or thrice as light as CD; it muſt have twice or thrice the hight, to have an equal preſſure, to counterbalance the other.

PROP. LXVII.

71. *If a body of the ſame ſpecific gravity of a fluid; be immerſed in it, it will reſt in any place of it. A body of greater denſity will ſink; and one of a leſs denſity will ſwim.*

Let A, B, C be three bodies; whereof A is lighter bulk for bulk than the fluid; B is equal; and C heavier. The body B, being of the ſame denſity, or equal in weight as ſo much of the fluid; it will preſs the fluid under it juſt as much as if the ſpace was filled with the fluid. The preſſure then will be the ſame all around it, as if the fluid was there, and conſequently there is no force to put it out of its place. But if the body be lighter, the preſſure of it downwards will be leſs than before; and leſs than in other places at the ſame depth; and conſequently the leſſer force will give way, and it will riſe to the top. And if the body be heavier, the preſſure downwards will be greater than before; and the greater preſſure will prevail and carry it to the bottom. Fig. 71.

Cor. 1. *Hence if ſeveral bodies of different ſpecific gravity be immerſed in a fluid; the heavieſt will get the loweſt.*

For the heavieſt are impelled with a greater force, and therefore will go faſteſt down.

Cor. 2. *A body immerſed in a fluid, loſes as much weight, as an equal quantity of the fluid weighs. And the fluid gains it.*

For if the body is of the ſame ſpecific gravity as the fluid; then it will loſe all its weight. And if it be lighter or heavier, there remains only the difference of the weights of the body and fluid, to move the body.

Cor. 3. *All bodies of equal magnitudes, loſe equal weights in the ſame fluid. And bodies of different magnitudes loſe weights proportional to the magnitudes.*

Cor. 4. *The weights loſt in different fluids, by immerging the ſame body therein, are as the ſpecific gravities of the fluids. And bodies of equal weight, loſe weights in the ſame fluid, reciprocally as the ſpecific gravities of the bodies.*

Cor. 5. *The weight of a body ſwimming in a fluid, is equal to the weight of as much of the fluid, as the immerſed part of the body takes up.*

For the preſſure underneath the ſwimming body is juſt the ſame as ſo much of the immerſed fluid; and therefore the weights are the ſame.

Cor.

Fig. 71. Cor. 6. *Hence a body will ſink deeper in a lighter fluid than in a heavier.*

Cor. 7. *Hence appears the reaſon why we do not feel the whole weight of an immerſed body, till it be drawn quite out of the water.*

PROP. LXVIII.

72. *If a fluid runs thro' a pipe, ſo as to leave no vacuities; the velocity of the fluid in different parts of it, will be reciprocally as the tranſverſe ſections, in theſe parts.*

Let AC, LB be the ſections at A and L. And let the part of the fluid ACBL come to the place *acbl*. Then will the ſolid ACBL = ſolid *acbl*; take away the part *ac*BL common to both; and we have AC*ca* = LB*bl*. But in equal ſolids the baſes and hights are reciprocally proportional. But if D*f* be the axis of the pipe, the hights D*d*, F*f*, paſſed thro' in equal times, are as the velocities. Therefore, ſection AC : ſection LB : : velocity along F*f* : velocity along D*d*.

PROP. LXIX.

73. *If* AD *is a veſſel of water or any other fluid*; B *a hole in the bottom or ſide. Then if the veſſel be always kept full; in the time a heavy body falls thro' half the hight of the water above the hole* AB, *a cylinder of water will flow out of the hole, whoſe hight is* AB, *and baſe the area of the hole.*

The preſſure of the water againſt the hole B, by which the motion is generated, is equal to the weight of a column of water whoſe hight is AB, and baſe the area B (by Cor. 2. Prop. LXVI.). But equal forces generate equal motions; and ſince a

cylinder of water falling thro' $\frac{1}{2}$ AB by its gravity, Fig. 73.
acquires ſuch a motion, as to paſs thro' the whole hight AB in that time. Therefore in that time the water running out muſt acquire the ſame motion. And that the effluent water may have the ſame motion, a cylinder muſt run out whoſe length is AB; and then the ſpace deſcribed by the water in that time will alſo be AB, for that ſpace is the length of the cylinder run out. Therefore this is the quantity run out in that time.

Cor. 1. *The quantity run out in any time is equal to a cylinder or priſm, whoſe length is the ſpace deſcribed in that time by the velocity acquired by falling thro' half the hight, and whoſe baſe is the hole.*

For the length of the cylinder is as the time of running out.

Cor. 2. *The velocity a little without the hole, is greater than in the hole; and is nearly equal to the velocity of a body falling thro' the whole hight* AB.

For without the hole the ſtream is contracted by the water's converging from all ſides to the center of the hole. And this makes the velocity greater in about the ratio of 1 to $\sqrt{2}$.

Cor. 3. *The water ſpouts out with the ſame velocity, whether it be downwards, or ſideways, or upwards. And therefore if it be upwards, it aſcends nearly to the hight of the water above the hole.*

Cor. 4. *The velocities and likewiſe the quantities of the ſpouting water, at different depths; will be as the ſquare roots of the depths.*

SCHOLIUM.

From hence are derived the rules for the conſtruction of fountains or jets. Let ABC be a reſervoir of water, CDE a pipe coming from it, to bring 74.

Fig. bring water to the fountain which ſpouts up at E, 74. to the hight EF, near to the level of the reſervoir AB. In order to have a fountain in perfection, the pipe CD muſt be wide, and covered with a thin plate at E with a hole in it, not above the fifth or ſixth part of the diameter of the pipe CD. And this pipe muſt be curve having no angles. If the reſervoir be 50 feet high, the diameter of the hole at E may be an inch, and the diameter of the pipe 6 inches. In general, the diameter of the hole E, ought to be as the ſquare root of the hight of the reſervoir. When the water runs thro' a great length of pipe, the jet will not riſe ſo high. A jet never riſes to the full hight of the reſervoir; in a 5 feet jet it wants an inch, and it falls ſhort by lengths which are as the ſquares of the hights; and ſmaller jets loſe more. No jet will riſe 300 feet high.

75. A ſmall fountain is eaſily made by taking a ſtrong bottle A, and filling it half full of water; cement a tube BI very cloſe in it, going near the bottom of the bottle. Then blow in at the top B, to compreſs the air within; and the water will ſpout out at B. If a fountain be placed in the ſunſhine and made to play, it will ſhew all the colours of the rainbow, if a black cloth be placed beyond it.

A jet goes higher if it is not exactly perpendicular; for then the upper part of the jet falls to one ſide without reſiſting the column below. The reſiſtance of the air will alſo deſtroy a deal of its motion, and hinder it from riſing to the hight of the reſervoir. Alſo the friction of the tube or pipe of conduct has a great ſhare in retarding the motion.

78. If there be an upright veſſel as AF full of water, and ſeveral holes be made in the ſide as B, C, D: then the diſtances, the water will ſpout, upon the horizontal plane EL, will be as the ſquare roots of the rectangles of the ſegments, ABE, ACE, and ADE. For the ſpaces will be as the velocities

Fig. 78. velocities and times. But (Cor. 4.) the velocity of the water flowing out of B, will be as $\sqrt{AB}$, and the time of its moving (which is the ſame as the time of its fall) will be (by Prop. XIII.) as $\sqrt{BE}$; therefore the diſtance EH is as $\sqrt{AB \times BE}$; and the ſpace EL as $\sqrt{ACE}$. And hence if two holes are made equidiſtant from top and bottom, they will project the water to the ſame diſtance, for if AB = DE, then ABE = ADE, which makes EH the ſame for both, and hence alſo it follows, that the projection from the middle point C will be furtheſt; for ACE is the greateſt rectangle. Theſe are the proportions of the diſtances; but for the abſolute diſtances, it will be thus. The velocity thro' any hole B, will carry it thro' 2AB in the time of falling thro' AB; then to find how far it will move in the time of falling thro' BE. Since theſe times are as the ſquare roots of the hights, it will be, $\sqrt{AB} : 2AB :: \sqrt{BE} : EH = 2AB\sqrt{\frac{BE}{AB}} = 2\sqrt{ABE}$; and ſo the ſpace EL $= 2\sqrt{ACE}$. It is plain, theſe curves are parabolas. For the horizontal motion being uniform; EH will be as the time; that is, as $\sqrt{BE}$, or BE will be as EH^2, which is the property of a parabola.

76. If there be a broad veſſel ABDC full of water, and the top AB fits exactly into it; and if the ſmall pipe FE of a great length be ſoldered cloſe into the top, and if water be poured into the top of the pipe F, till it be full; it will raiſe a great weight laid upon the top, with the little quantity of water contained in the pipe; which weight will be nearly equal to a column of the fluid, whoſe baſe is the top AB; and hight, that of the pipe EF. For the preſſure of the water againſt the top AB, is equal to the weight of that column of water,

Fig. 76. ter, by Prop. LXV. and Cor. And Prop. LXVI. Cor. 2.

But here the tube muſt not be too ſmall. For in capillary tubes the attraction of the glaſs will take off its gravity. If a very ſmall tube be immerſed with one end in a veſſel of water, the water will riſe in the tube above the ſurface of the water; and the higher, the ſmaller the tube is. But in quickſilver, it deſcends in the tube below the external ſurface, from the repulſion of the glaſs.

77. To explain the operation of a ſyphon, which is a crooked pipe CDE, to draw liquors off. Set the ſyphon with the ends C, E, upwards, and fill it with water at the end E till it run out at C; to prevent it, clap the finger at C, and fill the other end to the top, and ſtop that with the finger. Then keeping both ends ſtopt, invert the ſhorter end C into a veſſel of water AB, and take off the fingers, and the water will run out at E, till it be as low as C in the veſſel; provided the end E be always lower than C. Since E is always below C, the hight of the column of water DE is greater than that of CD, and therefore DE muſt out weigh CD and deſcend, and CD will follow after, being forced up by the preſſure of the air, which acts upon the ſurface of the water in the veſſel AB.

The ſurface of the earth falls below the horizontal level only an inch in 620 yards; and in other diſtances the deſcents are as the ſquares of the diſtances.

79. And to find the nature of the curve DCG, forming the jet IDG. Let AK be the hight or top of the reſervoir HF, and ſuppoſe the ſtream to aſcend without any friction, or reſiſtance. By the laws of falling bodies the velocity in any place B, will be as $\sqrt{AB}$. Put the ſemidiameter of the hole at D $= d$, and AD $= b$. Then ſince the ſame water paſſes thro' the ſections at D and B; therefore (Prop. LXVIII.) the velocity will be reciprocally

as the ſection; whence $\sqrt{h} : \frac{1}{dd} :: \sqrt{AB} : \frac{1}{BC^2}$; Fig. 79. therefore $\frac{\sqrt{h}}{BC^2} = \frac{\sqrt{AB}}{dd}$, and $dd\sqrt{h} = BC^2\sqrt{AB}$, whence $AB \times BC^4 = hd^4$; which is a paraboliform figure whoſe aſſymptote is AK, for the nature of the cataractic curve DCG. And if the fluid was to deſcend thro' a hole, as IC; it would form itſelf into the ſame figure GCD in deſcending.

PROP. LXX.

The reſiſtance any body meets with in moving thro' a fluid is as the ſquare of the velocity.

For if any body moves with twice the velocity of another body equal to it, it will ſtrike againſt twice as much of the fluid, and with twice the velocity; and therefore has four times the reſiſtance; for that will be as the matter and velocity. And if it moves with thrice the velocity, it ſtrikes againſt thrice as much of the fluid in the ſame time, with thrice the velocity, and therefore has nine times the reſiſtance. And ſo on for all other velocities.

Cor. *If a ſtream of water whoſe diameter is given, ſtrike againſt an obſtacle at reſt; the force againſt it will be as the ſquare of the velocity of the ſtream.*

For the reaſon is the ſame; ſince with twice or thrice the velocity; twice or thrice as much of the fluid impinges upon it, in the ſame time.

PROP. LXXI.

The force of a ſtream of water againſt any plane obſtacle at reſt, is equal to the weight of a column of water, whoſe baſe is the ſection of the ſtream; and hight, the ſpace deſcended thro' by a falling body, to acquire that velocity.

For let there be a reſervoir whoſe hight is that ſpace fallen thro'. Then the water (by Cor. 2. Prop.

Fig. Prop. LXIX.) flowing out at the bottom of the reſervatory, has the ſame motion as the ſtream; but this is generated by the weight of that column of water, which is the force producing it. And that ſame motion is deſtroyed by the obſtacle, therefore the force againſt it is the very ſame: for there is required as much force to deſtroy as to generate any motion.

Cor. *The force of a ſtream of water flowing out at a hole in the bottom of a reſervatory, is equal to the weight of a column of the fluid of the ſame hight and whoſe baſe is the hole.*

PROP. LXXII. *Prob.*

To find the ſpecific gravity of ſolids or fluids.

1. *For a ſolid heavier than water.*

Weigh the body ſeparately, firſt out of water, and then ſuſpended in water. And divide the weight out of water by the difference of the weights, gives the ſpecific gravity; reckoning the ſpecific gravity of water 1.

For the difference of the weights is equal to the weight of as much water (by Cor. 2. Prop. LXVII.); and the weights of equal magnitudes, are as the ſpecific gravities; therefore the difference of theſe weights; is to the weight of the body; as the ſpecific gravity of water 1; to the ſpecific gravity of the body.

2. *For a body lighter than water.*

Take a piece of any heavy body, ſo big as being tied to the light body, it may ſink it in water. Weigh the heavy body in and out of water, and find the loſs of weight. Alſo weigh the compound both in and out of water, and find alſo the loſs of weight. Then divide the weight of the light body (out of water), by the difference of theſe loſſes, gives the ſpecific gravity; the ſpecific gravity of water being 1.

For the laſt loſs is = weight of water equal in magnitude to the compound.

And the firſt loſs is = weight of water equal in magnitude to the heavy body.

Whence the dif. loſſes is = weight of water equal in magnitude to the light body.

and the weights of equal magnitudes, being as the ſpecific gravities; therefore the difference of the loſſes, (or the weight of water equal to the light body) : weight of the light body : : ſpecific gravity of water 1 : ſpecific gravity of the light body.

3. *For a fluid of any ſort.*

Take a piece of a body whoſe ſpecific gravity you know; weigh it both in and out of the fluid; take the difference of the weights, and multiply it by the ſpecific gravity of the ſolid body, and divide the product by the weight of the body (out of water), for the ſpecific gravity of the fluid.

For the difference of the weights in and out of water, is the weight of ſo much of the fluid as equals the magnitude of the body. And the weight of equal magnitudes being as the ſpecific gravities; therefore, weight of the ſolid : difference of the weights (or the weight of ſo much of the fluid) : : ſpecific gravity of the ſolid : to the ſpecific gravity of the fluid.

Example.

I weighed a piece of lead ore, which was 124 grains; and in water it weighed 104 grains, the difference is 20; then $\frac{124}{20} = 6.2$; the ſpecific gravity of the ore.

A table of ſpecific gravities.

Fine gold	19.640
Standard gold	18.888
Quickſilver	14.000
Lead	11.340
Fine ſilver	11.092
Standard ſilver	10.536
Copper	9.000
Copper half-pence	8.915
Gun metal	8.784
Fine braſs	8.350
Caſt braſs	8.100
Steel	7.850
Iron	7.644
Pewter	7.471
Tin	7.320
Caſt iron	7.000
Lead ore	6.200
Copper ore	5.167
Lapis calaminaris	5.000
Load ſtone	4.930
Antimony	4.000
Diamond	3.517
Iſland chriſtal	2.720
Stone, hard	2.700
Rock chriſtal	2.650
Glaſs	2.600
Flint	2.570
Common ſtone	2.500
Chriſtal	2.210
Brick	2.000
Earth	1.984
Horn	1.840

Ivory	1.820
Chalk	1.793
Allum	1.714
Clay	1.712
Oil of vitriol	1.700
Honey	1.450
Lignum vitæ	1.327
Treacle	1.290
Pitch	1.150
Rozin	1.100
Mohogany	1.063
Amber	1.040
Urine	1.032
Milk	1.031
Brazil	1.031
Box	1.030
Sea water	1.030
Ale	1.028
Vinegar	1.026
Tar	1.015
Common clear water	1.000
Bee wax	.955
Butter	.940
Linſeed oil	.932
Brandy	.927
Sallad oil	.913
Logwood	.913
Ice	.908
Oak	.830
Aſh	.830
Elm	.820
Oil of turpentine	.810
Walnut tree	.650
Fir	.580
Cork	.238
New fallen ſnow	.086
Air	.0012

Cor. 1. *As the weight loſt in a fluid, is to the abſolute weight of the body; ſo is the ſpecific gravity of the fluid, to the ſpecific gravity of the body.*

Cor. 2. *Having the ſpecific gravity of a body, and the weight of it; the ſolidity may be found thus; multiply the weight in pounds by* $62\frac{1}{2}$. *They ſay as that product to* 1; *ſo is the weight of the body in pounds, to the content in feet. And having the content given, one may find the weight, by working backwards.*

For a cubic foot of water weighs $62\frac{1}{2}$ lb. averdupoiſe; and therefore a cubic foot of the body weighs $62\frac{1}{2} \times$ by the ſpecific gravity of the body. Whence the weight of the body, divided by that product, gives the number of feet in it. Or as 1, to that product; ſo is the content, to the weight.

SCHOLIUM.

80. The ſpecific gravities of bodies may be found with a pair of ſcales; ſuſpending the body in water, by a horſe hair. But there is an inſtrument for this purpoſe called the *Hydroſtatical Balance*, the conſtruction of which is thus. AB is the ſtand and pedeſtal, having at the top two cheeks of ſteel, on which the beam CD is ſuſpended, which is like the beam of a pair of ſcales, and muſt play freely, and be it ſelf exactly in equilibrio. To this belongs the glaſs bubble G, and the glaſs bucket H, and four other parts E, F, I, L. To theſe are loops faſtened to hang them by. And the weights of all theſe are ſo adjuſted, that $E = F +$ the bubble in water, or $= I +$ the bucket out of water, or $= I + L +$ the bucket in water. Whence $L =$ difference of the weights of the bucket in and out of water. And if you pleaſe you may have a weight K, ſo that $K +$ bubble in water $=$ bubble out of water; or elſe find it in

grains. The piece L has a ſlit in it to ſlip it upon the ſhank of I. Fig. 80.

It is plain the weight K = weight of water as big as the bubble, or a water bubble.

Then to find the ſpecific gravity of a ſolid.

Hang E at one end of the balance, and I and the bucket with the ſolid in it, at the other end; and find what weight is a balance to it.

Then ſlip L upon I, and immerge the bucket and ſolid in the water, and find again what weight balances it. Then the firſt weight divided by the difference of the weights, is the ſpecific gravity of the body; that of water being 1.

For fluids.

Hang E at one end, and F with the bubble at the other; plunge the bubble into the fluid in the veſſel MN. Then find the weight P which makes a balance. Then the ſpecific gravity of the fluid is $= \frac{K+P}{K}$, when P is laid on F; or $= \frac{K-P}{K}$, when P is laid on E.

For E being equal to I + the bucket; the firſt weight found for a balance, is the weight of the ſolid. Again, E being equal to I + L + the bucket in water; the weight to balance that, is the weight of the ſolid in water; and the difference, is = to the weight of as much water. Therefore (Cor. 1.) the firſt weight divided by that difference, is the ſpecific gravity of the body.

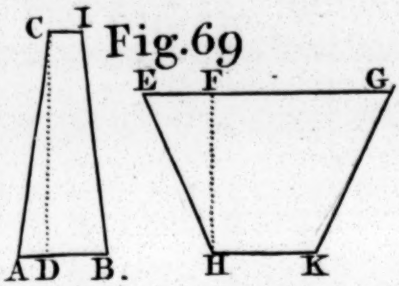
Again, ſince E is = to F + the bubble in water; therefore P is the difference of the weights, of the fluid and ſo much water; that is, P = difference of K and a fluid bubble; or P = fluid — K, when the fluid is heavier than water, or when P is laid on F. And therefore P = K — the fluid bubble, Fig. bubble, when contrary. Whence the fluid bubble 80. $= K \pm P$, for a heavier or lighter fluid. And the ſpecific gravities being as the weights of theſe equal bubbles; ſpecific gravity of water : ſpecific gravity of the fluid : : $K : K \pm P : : 1 : \frac{K \pm P}{K}$ the ſpecific gravity of the fluid. Where if P be 0, it is the ſame as that of water.

PROP. LXXIII.

The air is a heavy body, and gravitates on all parts of the ſurface of the earth.

That the air is a fluid is very plain, as it yields to any the leaſt force that is impreſſed upon it, without making any ſenſible reſiſtance. But if it be moved briſkly, by ſome very thin and light body, as a fan, or by a pair of bellows, we become very ſenſible of its motion againſt our hands or face, and likewiſe by its impelling or blowing away any light bodies, that lie in the way of its motion. Therefore the air being capable of moving other bodies by its impulſe, muſt it ſelf be a body; and muſt therefore be heavy like all other bodies, in proportion to the matter it contains; and will conſequently preſs upon all bodies placed under it. And being a fluid, it will dilate and ſpread itſelf all over upon the earth: and like other fluids will gravitate upon, and preſs every where upon its ſurface. The gravity and preſſure of the air is alſo evident from experiments. For (fig. 70.) if water, &c. be put into the tube ABF, and the air be drawn out of the end F by an air-pump, the water will aſcend in the end F, and deſcend in the end A, by reaſon of the preſſure at A, which was taken off or diminiſhed at F. There are numberleſs experiments of th's ſort. And tho' theſe properties

Fig. 69



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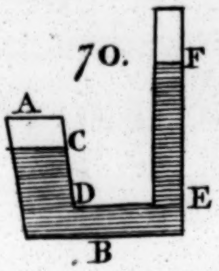

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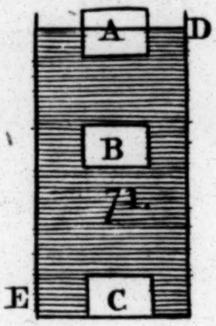

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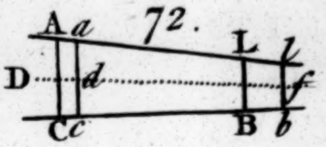

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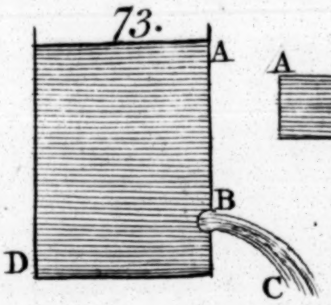

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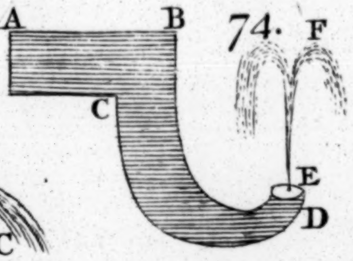

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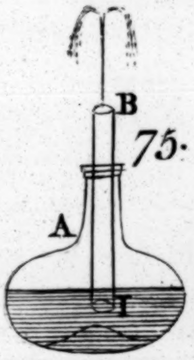

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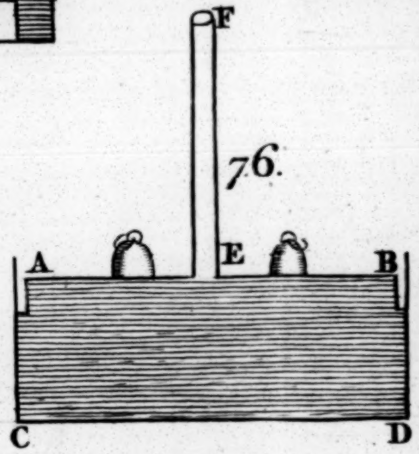

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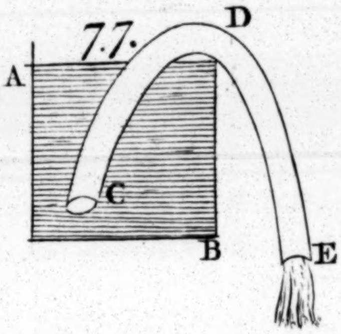

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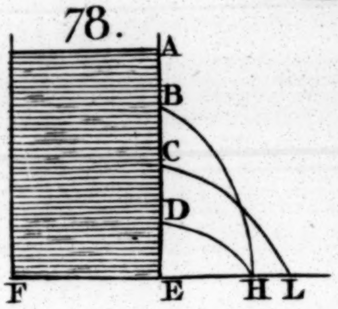

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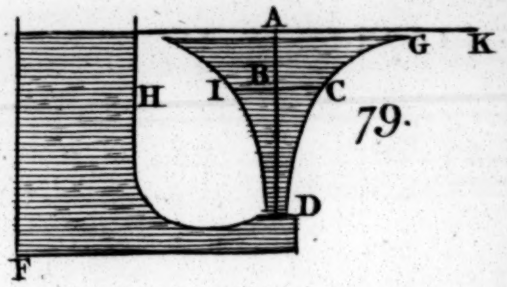

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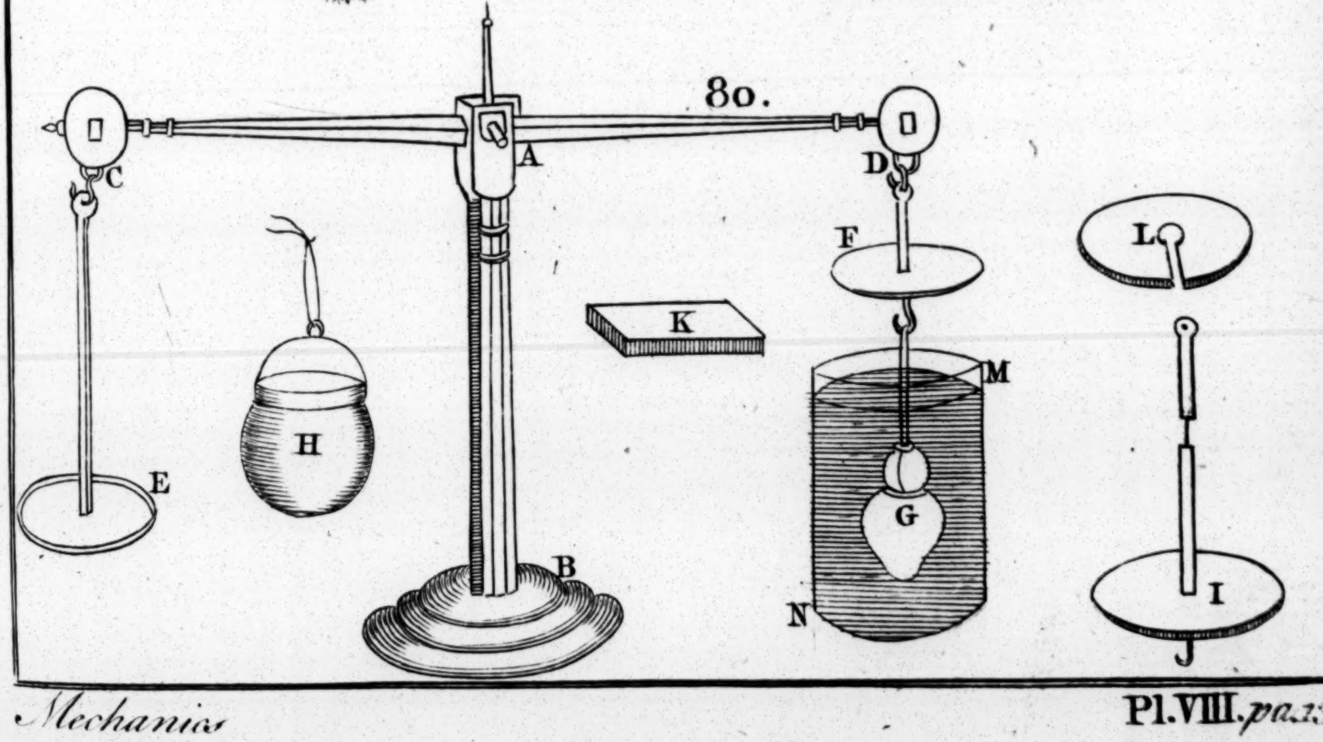


Mechanics

Pl. VIII. *pa. 134.*

perties and effects are certain, yet the air is a fluid ſo very fine and ſubtle, as to be perfectly tranſparent, and quite inviſible to the eye. Fig. 70.

Cor. 1. *The air, like other fluids, will, by its weight and fluidity, inſinuate itſelf into all the cavities, and corners within the earth; and there preſs with ſo much greater force, as the places are deeper.*

Cor. 2. *Hence the atmoſphere, or the whole body of air ſurrounding the earth, gravitates upon the ſurfaces of all other bodies, whether ſolid or fluid, and that with a force proportional to its weight or quantity of matter.*

For this property it muſt have in common with all other fluids.

Cor. 3. *Hence the preſſure, at any depth of water, or other fluid, will be equal to the preſſure of the fluid together with the preſſure of the atmoſphere.*

Cor. 4. *Likewiſe all bodies, near the ſurface of the earth, loſe ſo much of their weight, as the ſame bulk of ſo much air weight. And conſequently, they are ſomething lighter than they would be in a vacuum. But being ſo very ſmall it is commonly neglected; tho' in ſtrictneſs, the true or abſolute weight is the weight in vacuo.*

PROP. LXXIV.

The air is an elaſtic fluid, or ſuch a one, as is capable of being condenſed or expanded. And it obſerves this law, that its denſity is proportional to the force that compreſſes it.

Theſe properties of the air, are proved by experiments, of which there are innumerable. If you take a ſyringe, and thruſt the handle inwards, you'll feel the included air act ſtrongly againſt your hand;

Fig. hand; and the more you thruſt, the further the piſton goes in, but the more it reſiſts; and taking away your hand, the handle returns back to where it was at firſt. This proves its elaſticity, and alſo that air may be driven into a leſs ſpace, and condenſed.

75. Again, take a ſtrong bottle, and fill it half full of water, and cement a pipe BI, cloſe in it, going near the bottom; then inject air into the bottle thro' the pipe BI. Then the water will ſpout out at B, and form a jet; which proves, that the air is firſt condenſed, and then by its ſpring drives out the water, till it become of the ſame denſity as at firſt, and then the ſpouting ceaſes.

81. Likewiſe if a veſſel of glaſs AB be filled with water in the veſſel CD, and then drawn up with the bottom upwards; if any air is left in the top at A, the higher you pull it up, the more it expands; and the further the glaſs is thruſt down into the veſſel CD, the more the air is condenſed.

82. Again, take a crooked glaſs tube ABD open at the end A, and cloſe at D; pour in mercury to the hight BC, but no higher, and then the air in DC is in the ſame ſtate as the external air. Then pour in more mercury at A, and obſerve where it riſes to in both legs, as to G and H. Then you may always ſee that the higher the mercury is in the leg BH, the leſs the ſpace GD is, into which the air is driven. And if the hight of the mercury FH be ſuch as to equal the preſſure of the atmoſphere, then DG will be half DC; if it be twice the preſſure of the atmoſphere, DG will be $\frac{1}{3}$ DC, &c. So that the denſity is always as the weight or compreſſion. And here the part CD is ſuppoſed to be cylindrical.

Cor. 1. *The ſpace that any quantity of air takes up, is reciprocally as the force that compreſſes it.*

Cor. 2. *All the air near the earth is in a ſtate of compreſſion, by the weight of the incumbent atmoſphere.*

Cor. 3. *The air is denſer near the earth, or at the foot of a mountain, than at the top of it, and in high places. And the higher from the earth the more rare it is.*

Cor. 4. *The ſpring or elaſticity of the air is equal to the weight of the atmoſphere above it; and produces the ſame effects.*

For they always balance and ſuſtain each other.

Cor. 5. *Hence if the denſity of the air be increaſed; its ſpring or elaſticity will likewiſe be increaſed in the ſame proportion.*

Cor. 6. *From the gravity and preſſure of the atmoſphere, upon the ſurfaces of fluids, the fluids are made to riſe in any pipes or veſſels, when the preſſure within is taken off.*

PROP. LXXV.

The expanſion and elaſticity of the air is increaſed by heat, and decreaſed by cold. Or heat expands, and cold condenſes the air.

This is alſo matter of experience; for tie a bladder very cloſe with ſome air in it, and lay it before the fire, and it will viſibly diſtend the bladder; and burſt it if the heat is continued, and encreaſed high enough.

If a glaſs veſſel AB (Fig. 81.) with water in it, be turned upſide down, with a little air in the top A; and be placed in a veſſel of water, and hung over the fire, and any weight laid upon it to keep it down; as the water warms, the air in the top A, will by degrees expand, till it fills the glaſs, and by 81.

Fig. 81. by its elaſtic force, drive all the water out of the glaſs, and a good part of the air will follow, by continuing the veſſel there. Many more experiments may be produced proving the ſame thing.

PROP. LXXVI.

The air will preſs upon the ſurfaces of all fluids, with any force; without paſſing thro' them, or entering into them.

If this was not ſo, no machine, whoſe uſe or action depends upon the preſſure of the atmoſphere, could do its buſineſs. Thus the weight of the atmoſphere preſſes upon the ſurface of water, and forces it up into the barrel of a pump, without any air getting in, which would ſpoil its working. Likewiſe the preſſure of the atmoſphere keeps mercury ſuſpended at ſuch a hight, that its weight is equal to that preſſure; and yet it never forces itſelf thro' the mercury into the vacuum above, though it ſtand never ſo long. And whatever be the texture or conſtitution of that ſubtle inviſible fluid we call air, yet it is never found to paſs through any fluid, tho' it be made to preſs never ſo ſtrongly upon it. For tho' there be ſome air incloſed in the pores of almoſt all bodies, whether ſolid or fluid; yet the particles of air cannot by any force be made to paſs thro' the body of any fluid; or forced through the pores of it, although that force or preſſure be continued never ſo long. And this ſeems to argue that the particles of air are greater than the particles or pores of other fluids; or at leaſt are of a ſtructure quite different from any of them.

PROP.

PROP. LXXVII.

The weight or preſſure of the atmoſphere, upon any baſe at the earth's ſurface; is equal to the weight of a column of mercury of the ſame baſe, and whoſe hight is from 28 to 31 inches, ſeldom more or leſs.

This is evident from the barometer, an inſtrument which ſhews the preſſure of the air; which at ſome ſeaſons ſtands at a hight of 28 inches, ſometimes at 29, and 30, or 31. The reaſon of this is not, becauſe there is at ſome times more air in the atmoſphere, than at others; but becauſe the air being an extremely ſubtle and elaſtic fluid, capable of being moved by any impreſſions, and many miles high; it is much diſturbed by winds, and by heat and cold; and being often in a tumultuous agitation; it happens to be accumulated in ſome places, and conſequently depreſſed in others; by which means it becomes denſer and heavier where it is higher, ſo as to raiſe the column of mercury to 30 or 31 inches. And where it is lower, it is rarer and lighter, ſo as only to raiſe it to 28 or 29 inches. And experience ſhews, that it ſeldom goes without the limits of 28 and 31.

Cor. 1. *The air in the ſame place does not always continue of the ſame weight; but is ſometimes heavier, and ſometimes lighter; but the mean weight of the atmoſphere, is that when the quickſilver ſtands at about* 29 $\frac{1}{2}$ *inches.*

Cor. 2. *Hence the preſſure of the atmoſphere upon a ſquare inch at the earth's ſurface, at a medium, is very near 15 pounds, averdupoiſe.*

For an inch of quickſilver weighs 8.102 ounces.

Cor. 3. *Hence alſo the weight or preſſure of the atmoſphere, in its lighteſt and heavieſt ſtate, is equal to the*

Fig. *the weight of a column of water, 32 or 36 feet high; or at a medium 34 feet.*

For water and quickſilver are in weight nearly as 1 to 14.

Cor. 4. *If the air was of the ſame denſity to the top of the atmoſphere, as it is at the earth; its hight would be about 5 $\frac{1}{3}$ miles at a medium.*

For the weight of air and water are nearly as 12 to 1000.

Cor. 5. *The denſity of the air in two places diſtant from each other but a few miles, on the earth's ſurface and in the ſame level; may be looked on to be the ſame, at the ſame time.*

Cor. 6. *The denſity of the air at two different altitudes in the ſame place, differing only by a few feet; may be looked on as the ſame.*

Cor. 7. *If the perpendicular hight of the top of a ſyphon from the water, be more than 34 feet, at a mean denſity of the air. The ſyphon cannot be made to run.*

For the weight of the water in the legs will be greater than the preſſure of the atmoſphere, and both columns will run down, till they be 34 feet high.

Cor. 8. *Hence alſo the quickſilver riſes higher in the barometer, at the bottom of a mountain than at the top. And at the bottom of a coal pit, than at the top of it.*

SCHO-

SCHOLIUM.

Hence the denſity of the air may be found at any hight from the earth, as in the following table.

Miles	denſity	Miles	denſity
$\frac{1}{4}$	.9564	10	.1700
$\frac{1}{2}$	.9146	20	.02917
$\frac{3}{4}$	.8748	30	.005048
1	.8372	40	.000881
2	.7012	50	.000155
3	.5871	100	.0000000298
4	.4917		
5	.4119		

The firſt and third columns are the hight in miles from the ſurface of the earth. And the ſecond and fourth columns, ſhew the denſity at that hight; ſuppoſing the denſity at the ſurface of the earth, to be 1.

The denſity at any hight is eaſily calculated by this ſeries. Put $r =$ radius of the earth, $h =$ hight from the ſurface, both in feet. Then the denſity at the hight h, is the number belonging to the logarithm, denoted by this ſeries $-\frac{h}{68444} - \frac{h}{r}A - \frac{h}{r}B - \frac{h}{r}C$ &c. where A, B, C, &c. are the preceding terms. The terms here will be alternately negative and affirmative. But the firſt term alone is ſufficient when the hight is but a few miles.

By the weight and preſſure of the atmoſphere, the operations of pneumatic engines may be accounted for and explained. I ſhall juſt mention one or two.

A PUMP.

Fig. 83. is a *common pump*. AB the barrel or 83.
body of the pump, being a hollow cylinder, made of

Fig. 83. of wood or lead. CD the handle movable about the pin E. DF an iron rod moving about a pin D; this rod is hooked to the bucket or ſucker FG, which moves up and down within the pump. The bucket FG is hollow, and has a valve or clack L at the top opening upwards. H a plug fixed at the bottom of the barrel, being likewiſe hollow, and a valve at I opening alſo upwards. BK the bottom going into the well at K; the pipe below B need not be large, being only to convey the water out of the well into the body of the pump. The plug H muſt be fixed cloſe that no water can get between it and the barrel; and the ſucker FG, is to be armed with leather, to fit cloſe that no air or water can get thro' between it and the barrel.

When the pump is firſt wrought, or any time in dry weather when the water above the ſucker is waſted, it muſt be primed, by pouring in ſome water at the top A to cover the ſucker, that no air get through. Then raiſing the end C of the handle, the bucket F deſcends, and the water will riſe thro' the hollow GL, preſſing open the valve L. Then putting down the end C raiſes the bucket F, and the valve L ſhuts by the weight of the water above it. And at the ſame time the preſſure of the atmoſphere forces the water up thro' the pipe KB, and opening the valve I, it paſſes thro' the plug into the body of the pump. And when the ſucker G deſcends again, the valve I ſhuts, and the water cannot return, but opening the valve L, paſſes thro' the ſucker GL. And when the ſucker is raiſed again, the valve L ſhuts again, and the water is raiſed in the pump. So that by the motion of the piſton up and down, and the alternate opening and ſhutting of the two valves; water is continually raiſed into the body of the pump, and diſcharged at the ſpout M.

The diſtance KG, from the well to the bucket, Fig. 83. muſt not be above 32 feet; for the preſſure of the atmoſphere will raiſe the water no higher, and if it is more, the pump will not work. It is evident a pump will work better when the atmoſphere is heavy than when it is light, there being a twelfth or fifteenth part difference, at different times. And when it is lighteſt it is only equal to 32 feet. Wherefore the plug H muſt always be placed ſo low, as that the ſucker GL may be within that compaſs.

A BAROMETER.

Fig. 84. is a *Barometer*, or an inſtrument to meaſure the weight of the air. It conſiſts of a glaſs cone ABC hollow within, filled full of mercury, and hermetically ſealed at the end C, ſo that no air be left in it. When it is ſet upright, the mercury deſcends down the tube BC, into the bubble A, which has a little opening at the top A, that the air may have free ingreſs and egreſs. At the top of the tube C, there muſt be a perfect vacuum. This is fixed in a frame, and hung perpendicular againſt a wall. Near the top C, on the frame, is placed a ſcale of inches, ſhewing how high the mercury is in the tube BC, above the level of it in the bubble A, which is generally from 28 to 31 inches, but moſtly about 29 or 30. Along with the ſcale of inches, there is alſo placed a ſcale of ſuch weather as has been obſerved to anſwer the ſeveral hights of the quickſilver. Such a ſcale you have annexed to the 84th figure. In dividing the ſcale of inches, care muſt be taken to make proper allowance for the riſing or falling of the quickſilver in the bubble A, which ought to be about half full, when it ſtands at 29 $\frac{1}{2}$, which is the mean hight. For whilſt the quickſilver riſes an inch at C, it deſcends a little in the bubble A, and that deſcent

84.

Fig. 84. deſcent muſt be deducted, which makes the diviſions be ſomething leſs than an inch. Theſe inches muſt be divided into tenth parts, for the more exact meaſuring the weight of the atmoſphere. For the pillar of mercury in the tube is always equal to the weight of a pillar of the atmoſphere of the ſame thickneſs. And as the hight of the quickſilver increaſes or decreaſes, the weight of the air increaſes or decreaſes accordingly. The tube muſt be near 3 feet long, and the bore not leſs than $\frac{1}{5}$ or $\frac{1}{6}$ of an inch, in diameter, or elſe the quickſilver will not move freely in it.

By help of the barometer, the hight of mountains may be meaſured by the following table. In which the firſt column is the hight of the mountain, &c. in feet or miles; the ſecond the hight of the quickſilver; and the third the deſcent of the quickſilver in the barometer; and this at a mean denſity of the air.

Feet	High Barom.	Deſcent	Feet	High Barom.	Deſcent.
0	29.500				
100	29.400	.100	2600	27.028	2.472
200	29.301	.199	2700	26.938	2.562
300	29.203	.297	2800	26.848	2.652
400	29.105	.395	2900	26.758	2.742
500	29.007	.493	3000	26.668	2.832
600	28.910	.590	3100	26.578	2.922
700	28.812	.688	3200	26.489	3.011
800	28.716	.784	3300	26.400	3.100
900	28.619	.881	3400	26.311	3.189
1000	28.523	.977	3500	26.222	3.278
1100	28.428	1.072	3600	26.136	3.364
1200	28.332	1.168	3700	26.049	3.451
1300	28.237	1.263	3800	25.961	3.539
1400	28.143	1.357	3900	25.874	3.626
1500	28.048	1.452	4000	25.786	3.714
1600	27.954	1.546	4100	25.699	3.801
1700	27.860	1.640	4200	25.613	3.887
1800	27.766	1.734	4300	25.527	3.973
1900	27.672	1.828	4400	25.441	4.059
2000	27.579	1.921	4500	25.355	4.145
2100	27.487	2.013	4600	25.270	4.230
2200	27.394	2.106	4700	25.185	4.315
2300	27.302	2.198	4800	25.101	4.399
2400	27.210	2.290	4900	25.017	4.483
2500	27.119	2.381	5000	24.933	4.567

Fig. 84.

The Table continued in MILES.

Miles	H. Barom.	Deſcent	Miles	H. Barom.	Deſcent
0.	29.50				
0.25	28.21	1.29	3.25	16.57	12.93
0.50	26.98	2.52	3.50	15.85	13.65
0.75	25.80	3.70	3.75	15.16	14.34
1.	24.70	4.80	4.	14.50	15.00
1.25	23.62	5.88	4.25	13.87	15.63
1.50	22.60	6.90	4.50	13.27	16.23
1.75	21.62	7.88	4.75	12.70	16.80
2.	20.68	8.82	5.	12.15	17.35
2.25	19.78	9.72	5.25	11.62	17.88
2.50	18.93	10.57	5.50	11.12	18.38
2.75	18.11	11.39	5.75	10.64	18.86
3.	17.32	12.18	6.	10.18	19.32

This table is made from a table of the air's denſity, made as in Schol. Prop. LXXVII. And then multiplying all the numbers thereof by 29.5 the mean denſity of the air. For the denſity of the air at any height above the earth is as the weight of the atmoſphere above it, (by Prop. LXXIV.); and that is as the height of the mercury in the barometer.

A WATER BAROMETER.

85. A barometer may alſo be made of water as in fig. 85, which is a water barometer. AB is a glaſs tube open at both ends, and cemented cloſe in the mouth of the bottle EF, and reaching very near the bottom. Then warming the bottle at the fire, part of the air will fly out; then the end A is put into a veſſel of water mixed with cochineal, which will go thro' the pipe into the bottle as it grows cold. Then it is ſet upright; and the water

may be made to ſtand at any point C, by ſucking or blowing at A. And if this barometer be kept to the ſame degree of heat, by putting it in a veſſel of ſand, it will be very correct for taking ſmall altitudes; for a little alteration in the weight of the atmoſphere, will make the water at C riſe or fall in the tube very ſenſibly. But if it be ſuffered to grow warmer, the water will riſe too high in the tube, and ſpoil the uſe of it; ſo that it muſt be kept to the ſame temper. Fig. 85.

If a barometer was to be made of water put into an exhauſted tube, after the manner of quickſilver; it would require a tube 36 feet long or more; which could hardly find room within doors. But then it would go 14 times more exact than quickſilver; becauſe for every inch the quickſilver riſes, the water would riſe 14; from whence every minute change in the atmoſphere would be diſcernable.

And the water barometer above deſcribed will ſhew the variation of the air's gravity as minutely as the other, if the bottle be large to hold a great quantity of air. And in any caſe, by reducing the bottle (ſo far as the air is contained) to a cylinder; and put $D =$ diameter of the bottle, $d =$ diameter of the pipe, $p =$ height of air, $x =$ riſing in the pipe, all in inches. Then the height of a hill in feet will be nearly $\overline{1 + \frac{408dd}{pDD}} \times 71x$. And if $y =$ height of the hill or any aſcent, $Q = \frac{408dd}{pDD}$. Then $x = \frac{y}{\overline{1 + Q} \times 71}$ very near, at a mean denſity of the air.

A THERMOMETER.

Fig. 86. is a *thermometer*, or an inſtrument to meaſure the degrees of heat and cold. AB is a hollow 86.

Fig. 86. hollow tube near two foot long, with a ball at the bottom; it is filled with ſpirits of wine mixed with cochineal, half way up the neck; which done, it is heated very much, till the liquor fill the tube, and then it is ſealed hermetically at the end A. Then the ſpirit contracts within the tube as it cools. It is incloſed in a frame, which is graduated into degrees, for heat and cold. For hot weather dilates the ſpirit, and makes it run further up the tube; and cold weather on the contrary, contracts it, and makes it ſink lower in the tube. And the particular diviſions, ſhew the ſeveral degrees of heat and cold; againſt the principal of which, the words heat, cold, temperate, &c. are written.

They that would ſee more machines deſcribed, may conſult my large book of Mechanics, where he will meet with great variety.

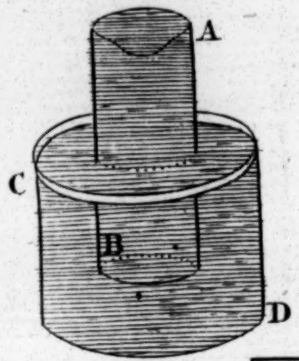
G.R. III

FINIS.

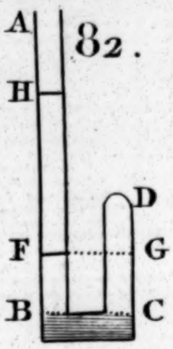
ERRATUM.

Page 35, Line 10 from the bottom, *read*, Cor. 1. Hence

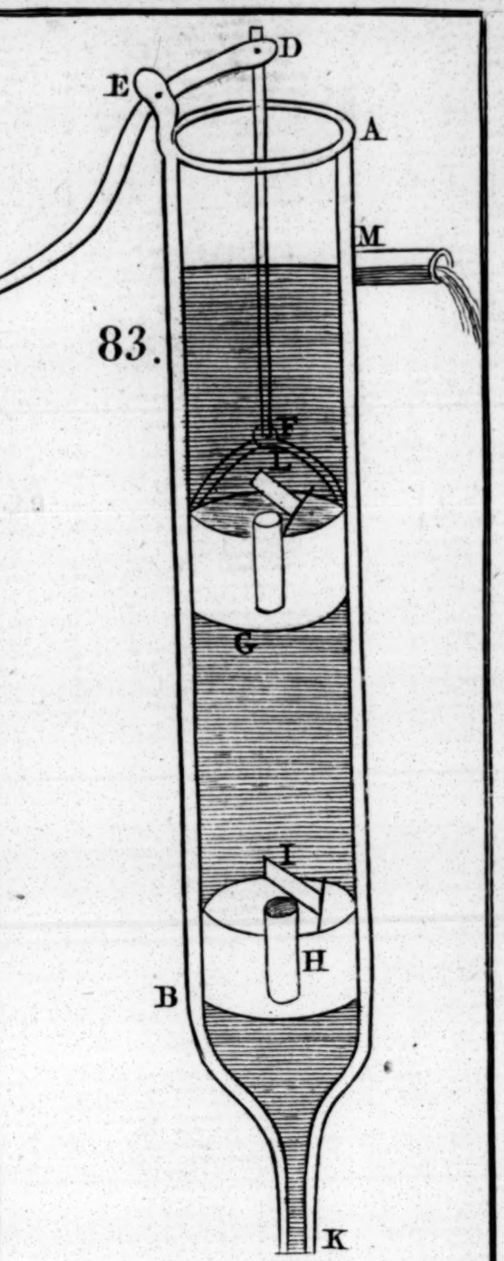
Fig. 81.



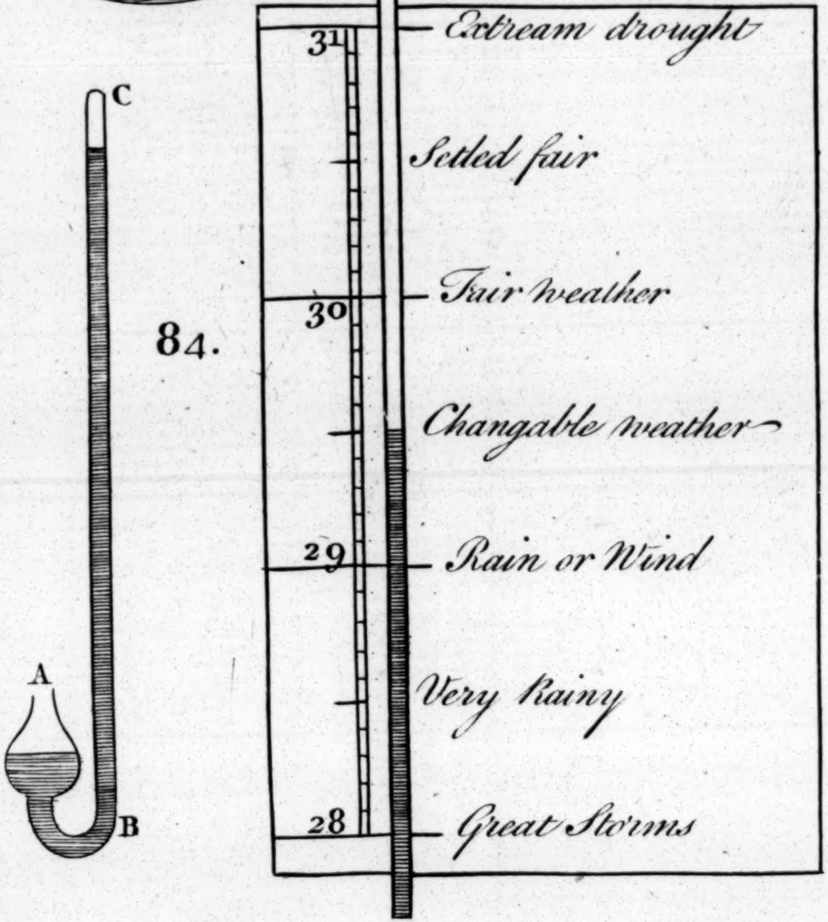
82.



83.



84.



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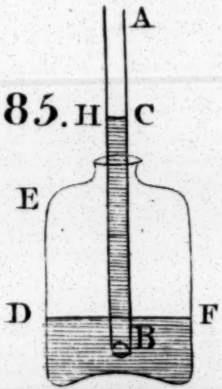

86.